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ACADÉMIE EUROPEENNE INTERDISCIPLINAIRE DES SCIENCES
INTERDISCIPLINARY EUROPEAN ACADEMY OF SCIENCES

<u>Lundi 4 décembre 2017:</u>
à 17 h à la Maison de l'AX, 5 rue Descartes 75005 PARIS

Conférence d'Armelle VIARD
Maître de Conférences à l'Ecole Pratique des hautes Etudes,
Chercheuse UMR_S 1077 INSERM-EPHE-UNICAEN, Caen:
"*Corrélats neuroanatomiques et neurofonctionnels de la mémoire humaine*"

<u>Notre Prochaine séance aura lieu le lundi 8 janvier 2018 à 15h45</u>
à l'Institut Henri Poincaré salle 01
11, rue Pierre et Marie Curie 75005 PARIS

Elle aura pour thème

Conférence de Marie AMALRIC ,
Chercheuse Post Doc au Département du Cerveau des Sciences Cognitives
Lab CAos/ Université de Rochester / Etat de New York/USA
"Comment le cerveau humain manipule-t-il les concepts mathématiques ?"

Académie Européenne Interdisciplinaire des Sciences
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décembre 2017

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TABLE DES MATIERES

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ACADEMIE EUROPEENNE INTERDISCIPLINAIRE DES SCIENCES INTERDISCIPLINARY EUROPEAN ACADEMY OF SCIENCES

5 rue Descartes 75005 PARIS

Séance du Lundi 4 décembre 2017 /Maison de l'AX 17h

La séance est ouverte à 17h **sous la Présidence de Victor MASTRANGELO** et en la présence de nos Collègues Gilbert BELAUBRE, Jean-Louis BOBIN, Juan-Carlos CHACHQUES, Gilles COHEN-TANNOUDJI, Claude ELBAZ, Michel GONDRAN, Irène HERPE-LITWIN, Gérard LEVY, Claude MAURY, Marie-Françoise PASSINI, Jacques PRINTZ, Jean SCHMETS , Jean-Pierre TREUIL, Alain STAHL,.

Etaient présents en tant que visiteurs Jean BERBINAU administrateur du lycée Saint Louis et du Collège Stanislas , Henri CABANAC, et René PUMAIN chercheur à l'INSEM.

Etaient excusés :François BEGON, Jean-Pierre BESSIS, Bruno BLONDEL, Michel CABANAC, Alain CARDON, Alain CORDIER , Daniel COURGEAU, Sylvie DERENNE, Ernesto DI MAURO, Jean-Felix DURASTANTI, Françoise DUTHEIL , Vincent FLEURY, Robert FRANCK, Jean -Pierre FRANÇOISE, Dominique LAMBERT, Valérie LEFEVRE-SEGUIN, Antoine LONG, Pierre MARCHAIS, Anastassios METAXAS, Jean-Jacques NIO, Alberto OLIVIERO, , Edith PERRIER, Pierre PESQUIES, Michel SPIRO, Mohand TAZEROUT , Jean-Paul TEYSSANDIER , Jean VERDETTI.

I. . **Présentation de notre conférencière Armelle VIARD par Victor MASTRANGELO**:

Armelle VIARD est depuis 2008 Maître de conférence et chercheuse à UMR_S 1077 INSERM-EPHE-UNICAEN (Université de Caen) dirigé par le Pr Francis EUSTACHE que nous avons accueilli récemment.

Précédemment elle a occupé les fonctions suivantes:

- **2006-2008**: Chercheur postdoctorant, University College London (UCL). Laboratoire: Institute of Cognitive Neuroscience, London, UK. Directeur: N. Burgess.

- **2005-2006**: Attaché Temporaire d'Enseignement et de Recherche (ATER), Université de Franche-Comté.

Ses diplômes universitaires sont les suivants:

- **2005**: Doctorat de neurosciences, Université Pierre et Marie Curie (UPMC, Paris VI).
- **2001**: Diplôme d'Etudes Approfondies en neurosciences, UPMC (Paris VI).
- **1999**: Maîtrise biologie cellulaire et physiologie, Université Denis Diderot (Paris VII).

Elle est l'auteur de 19 publications dans des revues à comité de lecture et son enseignement porte sur les domaines suivants:

- Neuroimagerie de la mémoire autobiographique (Master, EPHE).
- Systèmes de mémoire et vieillissement (Master, EPHE).
- Nosologie des maladies neurodégénératives et neurologiques (Master, EPHE).

- Amnésie et mémoire: neuropsychologie et neuroimagerie (Master, Université Paris VI).
- Neuropsychologie du langage (Licence, UNICAEN).

Elle est lauréate des prix suivants:

- **2004**: Prix de l'Association France Alzheimer.
- **2002**: Prix de l'Association France Alzheimer.

Elle est reviewer dans les revues internationales suivantes:

- Brain Imaging and Behaviour, BMC Neurology, British Journal of Clinical Psychology
- Experimental Aging Research, Frontiers in Behavioural Neuroscience
- Frontiers in Human Neuroscience
- Human Brain Mapping
- Journal of Neurophysiology Paris
- L'Année Psychologique
- Memory
- Neurobiology of Aging
- Neurocase, Neuropsychologia
- Proceedings of the Royal Society B: Biological Sciences
- Quarterly Journal of Experimental Psychology
- Revue de Neuropsychologie.

Ses 5 principales publications sont les suivantes:

- Viard, A., Piolino, P., Belliard, S., de La Sayette, V., Desgranges, B., & Eustache, F. (2014).Episodic future thinking in semantic dementia: a cognitive and fMRI study. *PLoS One*, 9, e111046.
- Viard, A., Desgranges, B., Matuszewski, V., Lebreton, K., Belliard, S., de La Sayette, V.Eustache, F., & Piolino, P. (2013). Autobiographical memory in semantic dementia: new insights from two patients using fMRI. *Neuropsychologia*, 51, 2620-2632.
- Viard, A., Doeller, C.F., Hartley, T., Bird, C.M., & Burgess, N. (2011). Anterior hippocampus and goal directed spatial decision making. *Journal of Neuroscience*, 31, 4613-4621.
- Viard, A., Lebreton, K., Chételat, G., Desgranges, B., Landeau, B., Young, A., de La Sayette,V., Eustache, F., & Piolino, P. (2010). Patterns of hippocampal-neocortical interactions in the retrieval of episodic autobiographical memories across the entire life-span of aged adults. *Hippocampus*, 20, 153-165.
- Viard, A., Piolino, P., Desgranges, B., Chételat, G., Lebreton, K., Landeau, B., Young, A., De La Sayette, V., Eustache, F. (2007). Hippocampal activation for autobiographical memories over the entire lifetime in healthy aged subjects: an fMRI study. *Cerebral Cortex*, 17, 2453-2467

II. Conférence d'Armelle VIARD

Résumé de la conférence avec références bibliographiques:

Corrélats neuroanatomiques et neurofonctionnels de la mémoire humaine par Armelle VIARD

Ce que nous appelons couramment "mémoire" s'applique à la mémoire épisodique , un terme inventé en 1972 par Endel Tulving. La mémoire épisodique est la mémoire d'événements autobiographiques intégrée dans un temps et une place spécifique , associée à des caractéristiques phénoménologiques telles que les émotions et les détails contextuels (qui, quoi, quand, où , pourquoi) . De ce fait, la mémoire autobiographique épisodique comporte un ensemble complexe d'opérations , incluant la réflexion sur soi (autoréflexion) , l'émotion, l'imagerie visuelle, l'attention, les fonctions de direction, et les processus sémantiques. L'imagerie cérébrale représente une contribution importante au développement de modèles de mémoire théoriques et de récentes avancées méthodologiques peuvent pister les nombreuses aires corticales associées au souvenir autobiographique mettant en évidence de fortes communications entre des aires corticales distantes. L'imagerie cérébrale a été utilisée aussi bien chez des sujets sains que chez des patients affectés par des troubles de la mémoire (maladie d'Alzheimer, démence sémantique, désordre post stress traumatique) . Des études récentes ont exploré le rôle de la mémoire épisodique dans la pensée future ou la capacité de nous projeter mentalement dans le futur en tant que moyen d'anticipation et d'évaluation des événements à venir avant qu'ils ne surviennent.

Références:

Tulving E (1972) Episodic and semantic memory. In Tulving E & Donaldson W (Eds) Organization of Memory. New York: Academic Press.

Eustache F, Viard A, Desgranges B (2016) The MNESIS model: memory systems and processes, identity and future thinking. Neuropsychologia 87:96-109.

Viard A, Piolino P, Belliard S, de La Sayette V, Desgranges B, Eustache F (2014) Episodic future thinking in semantic dementia: a cognitive and fMRI study. PLoS One 9: e111046.

Viard A, Desgranges B, Matuszewski V, Lebreton K, Belliard S, de La Sayette V, Eustache F, Piolino P (2013) Autobiographical memory in semantic dementia: new insights from two patients using fMRI. Neuropsychologia 51: 2620-2632.

Viard A, Lebreton K, Chételat G, Desgranges B, Landeau B, Young A, de La Sayette V, Eustache F, Piolino P (2010) Patterns of hippocampal-neocortical interactions in the retrieval of episodic autobiographical memories across the entire life-span of aged adults. Hippocampus 20: 153-165.

Viard A, Piolino P, Desgranges B, Chételat G, Lebreton K, Landeau B, Young A, De La Sayette V, Eustache F (2007) Hippocampal activation for autobiographical memories over the entire lifetime in healthy aged subjects: an fMRI study. Cerebral Cortex 17:2453-2467.

Sites:

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4652606/

Un compte-rendu détaillé sera prochainement disponible sur le site de l'AEIS , http://www.science-inter.com

Annonces

I. **Quelques ouvrages papiers relatifs au colloque de 2014 " Systèmes stellaires et planétaires- Conditions d'apparition de la Vie"** -

- Prix de l'ouvrage :25€ .
- Pour toute commande s'adresser à :

Irène HERPE-LITWIN Secrétaire générale AEIS
39 rue Michel Ange 75016 PARIS
06 07 73 69 75
irene.herpe@science-inter.com

II. **L'ouvrage cité ci-dessus est accessible gratuitement sur le site**:

http://www.edp-open.org/images/stories/books/fulldl/Formation-des-systemes-stellaires-et-planetaires.pdf

Documents

Pour introduire l'intervention de Marie AMALRIC nous vous proposons:

p. 08 : Un article de Marie Amalric et Stanislas Dehaene, S. *Origins of the brain networks for advanced mathematics in expert mathematicians*. PNAS, 04/2016; 113(18). DOI:10.1073/pnas.1603205113

Chapter 1. Origins of the brain networks for advanced mathematics in expert mathematicians

1. Introduction to the article

In this chapter, we introduce a novel paradigm to assess the brain representation of advanced mathematical concepts. For the first time, we have proposed to professional mathematicians to think about advanced mathematical problems while undergoing fMRI scanning. We chose to present mathematical problems from analysis, algebra, topology and geometry in linguistic format, through auditory sentences that were true, false, or meaningless. We compared mathematicians' reflection on mathematical statements with their reflection on control statements from nonmathematical domains such as history or geography. We also compared mathematicians to nonmathematician control subjects with similar academic standing but devoid of mathematical training beyond high school when they process advanced mathematical concepts as long as basic numerical processing.

2. Abstract

The origins of human abilities for mathematics are debated: some theories suggest that they are founded upon evolutionarily ancient brain circuits for number and space, others that they are grounded in language competence. To evaluate what brain systems underlie higher mathematics, we scanned professional mathematicians and mathematically naïve subjects of equal academic standing as they evaluated the truth of advanced mathematical and non-mathematical statements. In professional mathematicians only, mathematical statements, whether in algebra, analysis, topology or geometry, activated a reproducible set of bilateral frontal, intraparietal and ventrolateral temporal regions. Crucially, these activations spared areas related to language and to general-knowledge semantics. Rather, mathematical judgments were related to an amplification of brain activity at sites that are activated by numbers and formulas in non-mathematicians, with a corresponding reduction in nearby face responses. The evidence suggests that high-level mathematical expertise and basic number sense share common roots in a non-linguistic brain circuit.

3. Introduction

The human brain is unique in the animal kingdom in its ability to gain access to abstract mathematical truths. How this singular cognitive ability evolved in the primate lineage is currently unknown. According to one hypothesis, mathematics, like other cultural abilities that appeared suddenly with modern humans in the upper Paleolithic, is an offshoot of the human language faculty – for Noam Chomsky, for instance, "the origin of the mathematical capacity [lies in] an abstraction from linguistic operations" (Chomsky, 2006). Many mathematicians and physicists, however, disagree and insist that mathematical reflection is primarily non-linguistic – Albert Einstein, for instance, stated: « Words and language, whether written or spoken, do not seem to play any part in my thought processes. » (Hadamard, 1945).

An alternative to the language hypothesis has emerged from recent cognitive neuroscience research, according to which mathematics arose from an abstraction over evolutionarily ancient and non-linguistic intuitions of space, time, and number (Dehaene, 2011; Dillon et al., 2013). Indeed, even infants and uneducated adults with a drastically impoverished language for mathematics may possess abstract proto-mathematical intuitions of number, space and time (Dehaene et al., 2006; Pica et al., 2004). Such "core knowledge" is predictive of later mathematical skills (Gilmore et al., 2010; Halberda et al., 2008; Starr et al., 2013) and may therefore serve as a foundation for the construction of abstract mathematical concepts (Spelke, 2003). Advanced mathematics would arise from core representations of number and space through the drawing of a series of systematic links, analogies and inductive generalizations (Dehaene et al., 2008; Lakoff and Núñez, 2000; Piaget, 1952; Piaget and Inhelder, 1948).

The linguistic and core-knowledge hypotheses are not necessarily mutually exclusive. Linguistic symbols may play a role, possibly transiently, in the scaffolding process by which core systems are orchestrated and integrated (Carey, 2009; Spelke, 2003). Furthermore, mathematics encompasses multiple domains, and it seems possible that only some of them may depend on language. For instance, geometry and topology arguably call primarily upon visuospatial skills, while algebra, with its nested structures akin to natural language syntax, might putatively build upon language skills.

Contemporary cognitive neuroscience has only begun to investigate the origins of mathematical concepts, primarily through studies of basic arithmetic. Two sets of brain areas have been associated with number processing. Bilateral intraparietal and prefrontal areas are systematically activated during number perception and calculation (Dehaene et al., 1999), a circuit already present in infants and even in untrained monkeys (Nieder and Dehaene, 2009). Additionally, a bilateral inferior temporal region is activated by the sight of number symbols such Arabic numerals, but not by visually similar letters (Shum et al., 2013). Those regions lie outside of classical language areas, and several fMRI studies have confirmed a double dissociation between the areas involved in number sense and language (Cantlon and Li, 2013; Monti et al., 2012). Only a small part of our arithmetic knowledge, namely the rote memory for arithmetic facts encoded in linguistic form (Dehaene et al., 1999; Spelke and Tsivkin, 2001). The bulk of number comprehension and even algebraic manipulations can remain preserved in patients with global aphasia or semantic dementia (Cappelletti et al., 2012; Lemer et al., 2003; Varley et al., 2005). Contrary to intuition, brain-imaging studies of the processing of nested arithmetic expressions show little or no overlap with language areas (Friedrich and Friederici, 2009; Maruyama et al., 2012; Nakai and Sakai, 2014). Thus, conceptual understanding of arithmetic, at least in adults, seems independent of language.

Many mathematicians, however, argue that number concepts are too simple to be representative of advanced mathematics. To address this criticism, here we study the cerebral representation of high-level mathematical concepts in professional mathematicians. We collected functional magnetic resonance images (fMRI) in 15 professional mathematicians and 15 non-mathematicians controls of equal academic standing, while participants performed fast semantic judgments on mathematical and non-mathematical statements (figure 1.1A). On each trial, a short spoken sentence was followed by a 4-second reflection period during which the participants decided whether the statement was true, false or meaningless. Meaningful and meaningless statements were matched on duration and lexical content, but meaningless statements could be quickly dismissed, while meaningful statements required in-depth thinking, thus presumably activating brain areas involved in conceptual knowledge. Statements were generated with the help of professional mathematicians and probed four domains of higher mathematics: analysis, algebra, topology, and geometry. A fifth category of non-math sentences, matched in length and complexity, probed general knowledge of nature and history. Two additional fMRI runs evaluated sentence processing and calculation (Pinel et al., 2007) and the visual recognition of faces, bodies, tools, houses, numbers, letters, and written mathematical expressions.

4. Methods

4.1. Participants

We scanned a total of 30 French adult participants. 15 were professional mathematicians (11 male, 4 female, age range 24-39, mean = 28.1) and 15 were humanities specialists (10 male, 5 female, age range 24-50, mean = 30.1). Their ages did not significantly differ ($t = 0.8397$, $p = 0.41$).

Professional mathematicians were full-time researchers and/or professors in mathematics. All had a PhD in Mathematics and/or had passed the French national examination called “aggregation” which is the last qualification exam for professorship. The 15 control subjects had the same education level, but had specialized in humanities and had never received any mathematical courses since high school. Their disciplines were: literature (n = 3), history (n = 3), philosophy (n = 1), linguistics (n = 2), antiquity (n = 1), graphic arts and theatre (n = 3), communication (n = 1) and heritage conservation (n = 1). All subjects gave written informed consent and were paid for their participation. The experiment was approved by the regional ethical committee for biomedical research.

4.2. Visual runs

Seven categories of images were presented: faces, houses, tools, bodies, words, numbers, and mathematical formulas, plus a control condition consisting of circular checkerboards whose retinotopic extend exceeded that of all other stimuli.

All stimuli were black on a white background. Faces, tools, houses and bodies were highly contrasted gray-level photographs matched for overall number of gray level. Faces were front or slightly lateral views of non-famous people. Houses consisted in outside views of houses or buildings. Tools were common hand-held household object such as a hair-dryer. Bodies were front pictures of headless standing bodies. Numbers, words and formulas were strings of 5 or 6 characters. All numbers were decimal forms of famous constants (e.g. 3.14159 = π). Formulas were extracted from classical mathematical equations or expressions (e.g. binomial coefficients or the Zeta function). Words were written either with upper or lower case letters and were of high lexical frequency (mean = 28.3 per million; http://lexique.org).

Although numbers, words and formulas were inevitably arranged horizontally relative to other images, the mean width of horizontal images was not significantly different from the mean length of vertical images or the mean side of the square ones, so that they were all inscribed in a circle of 310 pixels diameter, equivalent to a visual angle of 5°.

The stimuli were presented in short mini-blocks of eight stimuli belonging to the same category. Within each block, the subject's task was to click a button whenever he/she detected an image repetition (one-back task). Each of the seven categories of images comprised twelve items, among which eight items were randomly picked on a given mini-block. Each image was flashed for 300 ms and followed by a 300 ms fixation point, for a total duration of 4.8 s. The category blocks were separated by a brief resting period with a fixation point only, whose duration was randomly picked among 2.4 s, 3.6 s or 4.8 s.

4.3. Auditory runs

Subjects were presented with 72 mathematical statements (18 in each of the fields of analysis, algebra, topology and geometry) and 18 non-mathematical statements. Within each category, 6 statements were true, 6 were false, and 6 were meaningless. All meaningless statements (in math or non-math) were grammatically correct but consisted in meaningless associations of words extracted from unrelated meaningful statements. All meaningful statements bore upon non-trivial facts which were judged unlikely to be stored in rote long-term memory and therefore required logical reflection. Reference to numbers or to other mathematical concepts (e.g. geometrical shapes) was purposely excluded. A complete list of statements, translated from the original French, is presented in appendix.

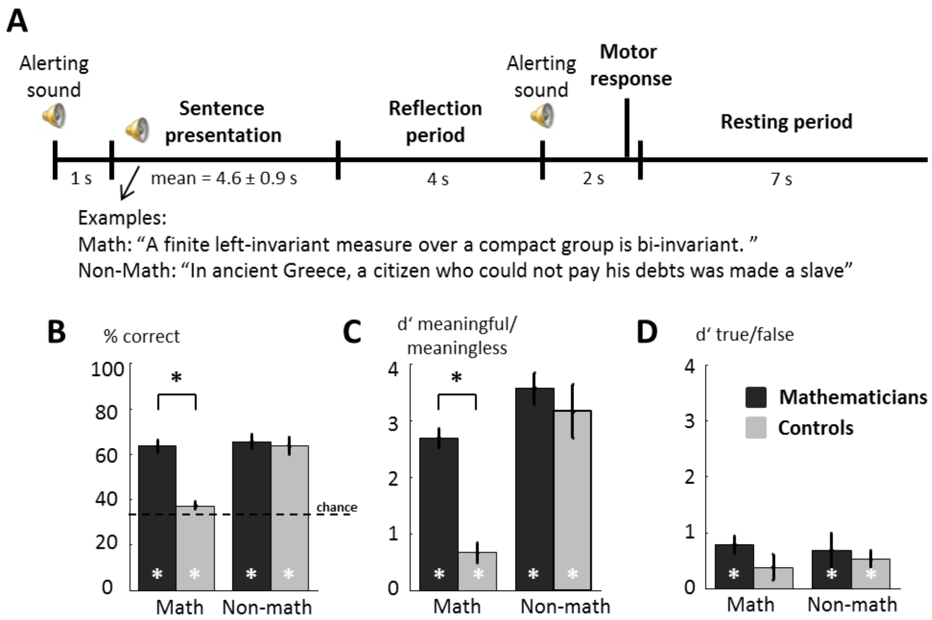

Figure 1.1. Main paradigm and behavioral results. (A) On each trial, subjects listened to a spoken statement and, four seconds later, classified it as true, false or meaningless. (B) Performance in this task (% correct). (C, D) Mean d-prime values for discrimination of meaningful versus meaningless statements (C) and, within meaningful statements, of true versus false statement (D). *, $p < 0.05$ (student t-tests). Error bars represent one standard error of the mean (SEM).

All statements were recorded by a female native French speaker who was familiar with mathematical concepts. Statements from the different categories were matched in syntactic construction, length (mean number of words: math = 12.4, non-math = 12.6, $t = 0.24$, $p = 0.81$) and duration (mean duration in s: math = 4.70, non-math = 4.22, $t = 1.93$, $p = 0.056$).

The experiment was divided into 6 runs of 15 statements each, which included one exemplar of each sub-category of statements (5 categories [analysis, algebra, geometry, topology, or general knowledge] x 3 levels [true, false, or meaningless]). On screen, the only display was a fixation cross on a black background. Each trial started with a beep and a color change of the fixation cross (which turned to red), announcing the onset of the statement. Following auditory presentation, a fixed-duration reflection period (4 seconds) allowed subjects to decide whether the statement was true, false or meaningless. The end of the reflection period was signaled with a beep and the fixation cross turning to green. Only then, for 2 seconds, could subjects give their evaluation of the sentence (true, false, or meaningless) by pressing one of three corresponding buttons (held in the right hand). Each trial ended with a 7-second resting period (figure 1.1A).

4.4. Localizer scan

This 5-minute fMRI scan is described in detail elsewhere (Pinel et al., 2007). For present purposes, only two contrasts were used: language processing (sentence reading + sentence listening relative to rest) and mental calculation (mental processing of simple subtraction problems such as 7-2, presented visually or auditory, and contrasted to the processing of non-numerical visual or auditory sentences of equivalent duration and complexity).

4.5. Post-MRI questionnaire

Immediately after fMRI, all the statements that had been presented during fMRI were reexamined in the same order. For each of them, participants were asked to rate their comprehension of the problem itself within the noisy environment of the fMRI machine; their confidence in their answer; whether the response was a well-known fact or not (variable hereafter termed “immediacy”); the difficulty of the statement; its “imageability”; and the kind of reasoning that they had used on an axis going from pure intuition to the use of a formal proof.

4.6. fMRI data acquisition and analysis

We used a 3-Tesla whole body system (Siemens Trio) with a 32 channel head-coil and high-resolution multiband imaging sequences developed by the Center for Magnetic Resonance Research (CMRR) (Xu et al., 2013) (multiband factor = 4, Grappa factor = 2, 80 interleaved axial slices, 1.5 mm thickness and 1.5 mm isotropic in-plane resolution, matrix = 128x128, TR = 1500 ms, TE = 32 ms).

Using SPM8 software, functional images were first realigned, normalized to the standard MNI brain space, and spatially smoothed with an isotropic Gaussian filter of 2 mm FMWH.

A two-level analysis was then implemented in SPM8. For each participant, fMRI images were high-pass filtered at 128s. Then, time series from visual runs were modelled by regressors obtained by convolution of the 8 categories of pictures plus the button presses with the canonical SPM hemodynamic response function (HRF) and its time derivative. Data from the auditory runs was modelled by two regressors for each sentence, one capturing the activation to the sentence itself (kernel = sentence duration) and the other capturing the activation during the reflection period (4-s rectangular kernel). We then defined subject-specific contrasts over specific sentences, either comparing the activation evoked by any two subsets of sentences (during sentence presentation or during the post-sentence reflection period), or evaluating the impact of a continuous variable such as subjective difficulty on a subset of sentences. Regressors of non-interest included the six movement parameters for each run. Within each auditory run, two additional regressors of non-interest were added to model activation to the auditory beeps and to the button presses.

For the second-level group analysis, individual contrast images for each of the experimental conditions relative to rest were smoothed with an isotropic Gaussian filter of 5 mm FWHM, and separately for visual and auditory runs, entered into a second-level whole-brain ANOVA with stimulus category as within-subject factor. All brain-activation results are reported with a clusterwise threshold of $p < 0.05$ corrected for multiple comparisons across the whole brain, using an uncorrected voxelwise threshold of $p < 0.001$.

5. Results

5.1. Behavioral results

5.1.1. Behavioral results in auditory runs

With mathematical statements, mathematicians performed way above chance level (63.6 ± 2.8 % [mean ± standard error]; chance = 33.3%; Student's t test, $t = 11.3$ $p < 0.001$, figure 1.1B), while control subjects unsurprisingly fell close to chance level (37.4 ± 1.6 %, $t = 2.6$, $p = 0.02$; difference between groups: $t = 8.5$, $p< 0.001$). With non-mathematical statements, both groups performed equally well (mathematicians: 65.4 ± 3.1 %, $t = 10.6$, $p < 0.001$; controls: 63.7 ± 3.8 %, $t = 8.3$, $p < 0.001$; no difference between groups: $t = 0.4$, $p = 0.7$). Importantly, mathematicians performed identically with math and non-math statements ($t = 0.5$, $p = 0.6$), thus suggesting that math and non-math problems were well-matched in objective difficulty level.

Above-chance performance could arise from a discrimination of meaningful and meaningless statements, from a discrimination of true versus false statements, or both. To separate these effects, we applied signal detection theory (SDT). First, we quantified subjects' ability to discriminate whether the statements were meaningful (pooling across true and false statements) or meaningless. We considered hits as "meaningful" responses to statements that were indeed meaningful, and false alarms as "meaningful" responses to meaningless statements. For both mathematics and non-mathematics, mathematicians' judgments of meaningfulness were highly above chance ($d'_{math} = 2.68 \pm 0.18$, $t = 15.9$, $p < 0.001$; $d'_{non\text{-}math} = 3.56 \pm 0.28$, $t = 13.0$, $p < 0.001$). On the contrary, controls' judgments of meaningfulness dropped nearly to 0 for mathematics ($d'_{math} = 0.67 \pm 0.17$, $t = 3.9$, $p = 0.002$), but were highly above chance for general knowledge ($d'_{non\text{-}math} = 3.16 \pm 0.47$, $t = 6.99$, $p < 0.001$). There was no significant difference comparing mathematicians and controls' capacity to discriminate meaningful non mathematical sentences ($t = 0.76$, $p = 0.45$). However, mathematicians were significantly better than controls at discriminating meaningful mathematical statements ($t = 8.44$, $p < 0.001$) (figure 1.1C).

We also applied SDT to evaluate the subjects' capacity to discriminate true and false statements. This analysis was restricted to meaningful statements that were judged meaningful. We considered hits as true statements correctly classified as true, and false alarms as false statements incorrectly classified as true. Mathematicians showed weak but significantly positive d-primes for mathematics ($d'_{math} = 0.78 \pm 0.16$, $t = 5.0$, $p < 0.001$), and for non-mathematics ($d'_{non\text{-}math} = 0.68 \pm 0.31$, $t = 2.30$, $p = 0.04$). Controls did not show a significantly positive d-prime for mathematics but they did for non-mathematics ($d'_{math} = 0.38 \pm 0.23$, $t = 1.72$, $p = 0.11$; $d'_{non\text{-}math} = 0.52 \pm 0.15$, $t = 3.48$, $p = 0.004$). The difference between mathematicians and controls failed to reach significance, either for mathematics ($t = 1.46$, $p = 0.15$) or for general knowledge ($t = 0.49$, $p = 0.63$) (figure 1.1D).

In summary, mathematicians performed equally well with both types of sentences. Within the allotted time period of 4 seconds, they managed to discriminate meaningful mathematical statements from meaningless ones, as well as to distinguish true statements from false ones. Controls only managed to understand and classify the non-mathematical sentences. Most importantly, the results indicate that mathematical statements and non-mathematical sentences were well matched in term of objective difficulty, as evaluated by percent success, and that mathematicians and control subjects were well matched in terms of their performance with non-mathematical statements.

5.1.2. Behavioral results in visual runs

SDT was also used to evaluate subjects' ability to perform the visual one-back task. Pooling across the groups, *d's* for each category were significantly greater than 0 (minimum d' averaged across subjects = 2.4, all $p < 10^{-12}$), meaning that participants correctly detected repetitions within each visual category. An ANOVA on *d's*, with category as a within-subject factor and group as a between-subjects factor, indicated that neither mathematical expertise nor the category of pictures influenced the performance, and that both groups performed equally well in detecting repetitions regardless of the visual category (group: $F = 0.18$, $p = 0.67$; category: $F = 0.29$, $p = 0.94$; interaction group x category: $F = 0.69$, $p = 0.66$). An ANOVA on reaction time showed equivalent results (group: $F = 1.63$, $p = 0.20$; category: $F = 0.67$, $p = 0.67$; interaction group x category: $F = 0.54$, $p = 0.78$). Obviously, the one-back task was simple enough that, in spite of their mathematical expertise, mathematicians performed no better than controls in detecting repetitions, even with numbers ($t = 0.83$, $p = 0.41$) or formulas ($t = 0.83$, $p = 0.41$).

5.1.3. Subjective variables reported during the post-MRI questionnaire

For mathematical statements, mathematicians gave higher ratings than controls for all subjective variables (all *p*s < 0.001) (figure S0). For non-mathematical sentences, ratings of understanding, immediacy and imageability were equivalent for both groups, and controls responded with higher ratings than mathematicians for confidence, ease of responding, and reflection (*p*s < 0.05). Those findings suggest that each group was more at ease with its respective domain of expertise (figure S0).

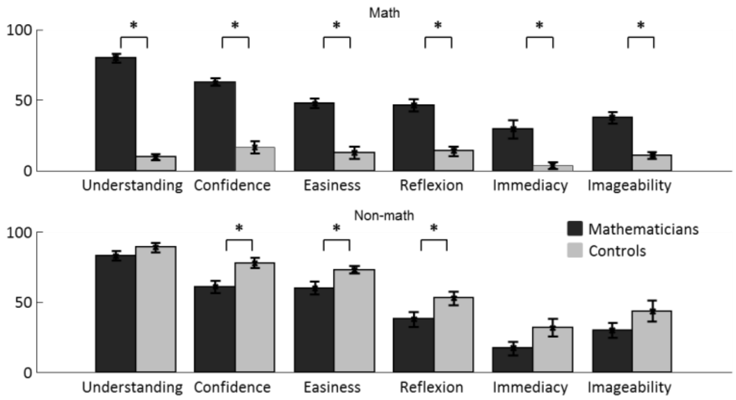

Figure S0. Participants' subjective ratings. Subjective ratings of understanding, confidence, ease of responding, intuition, immediacy and imageability for math (top) and nonmath (bottom) statements in both mathematicians (black) and control subjects (gray).

To evaluate the reliability of subjective ratings, which were collected after the fMRI, we correlated them with objective performance to the same statements. Within the group of professional mathematicians, we observed that objective performance during fMRI was positively correlated with subsequent ratings of confidence (logistic regression, $r = 0.36$; $p < 0.001$) and comprehension ($r = 0.21$; $p < 0.001$) of the same statements, and negatively correlated with subjective difficulty ($r = -0.28$; $p < 0.001$) and intuition ($r = -0.11$; $p < 0.001$). Those relations indicate that subjective variables were reliable and that, unsurprisingly perhaps, mathematicians showed increasingly better performance on sentences that they understood better, rated as easier, were more confident about, and for which they deployed explicit reasoning rather than mere intuitive judgments.

5.2. fMRI activations associated with mathematical reflection

Within the group of professional mathematicians, we first searched for greater activations to math than to non-math judgments during the reflection period. This contrast identified an extensive set of areas involving the bilateral intraparietal sulci (IPS), bilateral inferior temporal (IT) regions, bilateral dorsolateral, superior and mesial prefrontal cortex (PFC), and cerebellum (figures 1.2 and S1; table S1).

Examination of the time course of activity indicated that, at all sites of the shared math network, the fMRI signal rose sharply after a mathematical statement and remained sustained for ~15 seconds (figures 1.2C and S1).

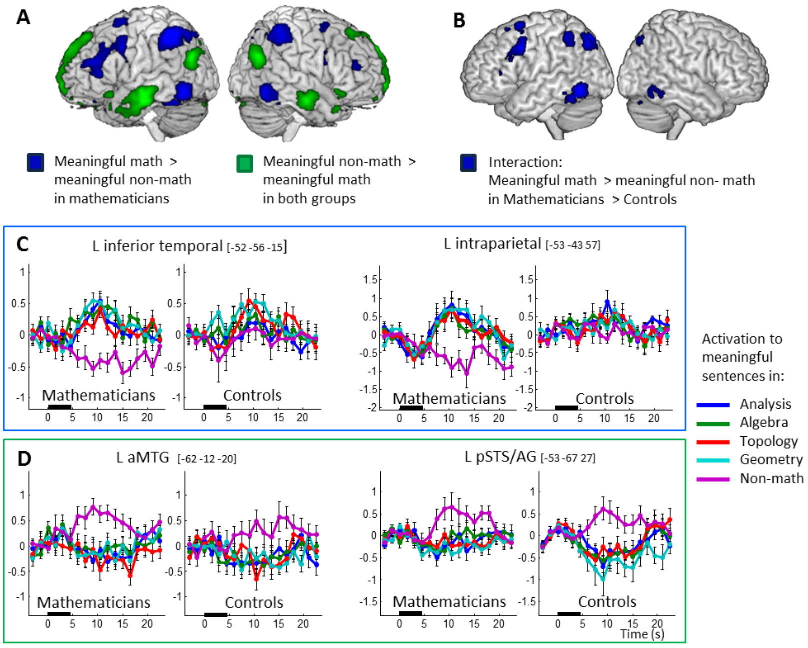

Figure 1.2. Distinct brain areas for mathematical expertise and for general semantic knowledge. (A) Whole-brain view of areas activated during reflection on mathematical statements (blue) versus general knowledge (green). In this figure and all subsequent figures, brain maps are thresholded at voxel $P < 0.001$, cluster $P < 0.05$ corrected for multiple comparisons across the brain volume. (B) Mathematical expertise effect: Interaction indicating a greater difference between meaningful math and nonmath statements in mathematicians than in controls. (C and D) Average fMRI signals in representative areas responsive to math (C) and to nonmath (D) (see SI Appendix, Fig. S1 for additional areas). Black rectangles indicate sentence presentation.

Contrariwise, for non-mathematical statements, a slow deactivation was seen (figure 1.2C). Thus, this network was strongly activated by all domains of mathematics, but remained inactive during reflection on matched non-mathematical problems. Furthermore, an interaction with group (math>non-math X mathematicians>controls) showed that this activation pattern was unique to subjects with mathematical expertise (figure 1.2B, table S1).

In control subjects, the math > non-math contrast identified a set of cortical areas involving right pre-central and left postcentral sulci, bilateral mesial parietal, middle occipital gyri, lingual gyri, insula overlapping with BA13, different frontal sites in BA10, parts of orbitofrontal prefrontal cortex and middle frontal gyrus, and subcortical regions, especially bilateral putamen (Figure S2A, Table S1). Those activations partly resemble the activations evoked by meaningless general-knowledge statements. Indeed, the meaningless > meaningful non-math contrast revealed activations in the right supramarginal gyrus, bilateral mesial parietal, right lingual gyrus, left anterior superior temporal gyrus (aSTG), near temporal pole, right pre-central and left post-central sulci. Activation maps for these two contrasts overlapped in the right pre-central and left post-central sulci, bilateral mesial parietal and right lingual gyrus (figure S2B). In aSTG, we observed a strong deactivation for meaningless non-math and no activation for math (figure S2C).

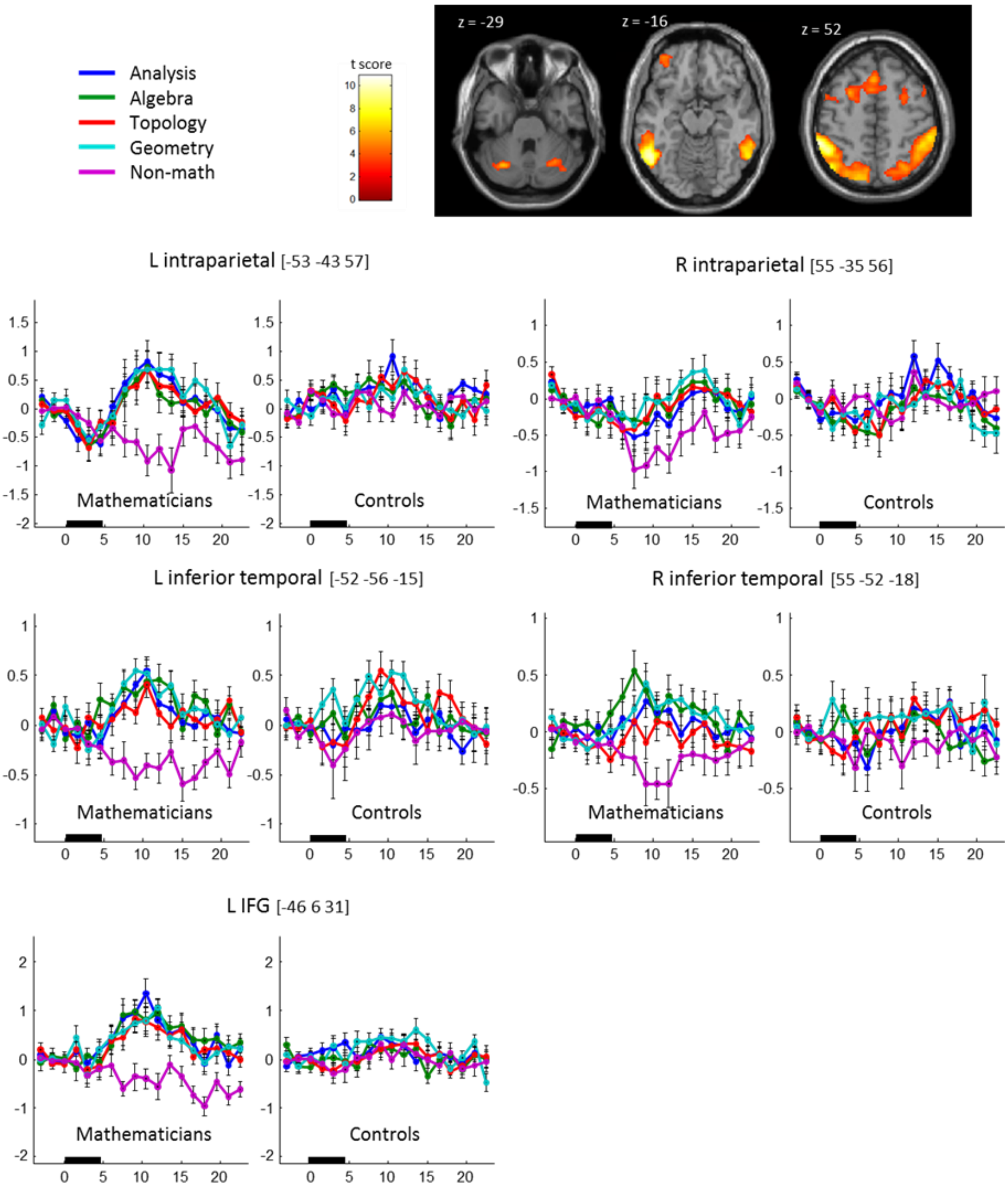

Figure S1. Activation profiles in areas activated by mathematical reflection in professional mathematicians. (Top) Axial slices showing voxels where activation was higher during reflection on math statements relative to non-math statements (voxel $p < 0.001$, cluster $p < 0.05$ corrected for multiple comparisons at the whole-brain level). (Bottom) Plots show the fMRI signal (mean +/- one standard error) at the main peak of the main significant clusters. Time scale starts 3 seconds before the presentation of the sentence and lasts until the end of a trial. Black rectangles indicate the approximate time of sentence presentation.

These results suggest that control subjects, when listening to mathematical statements (1) do not activate the same bilateral intraparietal and inferior temporal regions as professional mathematicians; and (2) process both meaningful and meaningless mathematical statements in a manner similar to meaningless non-mathematical statements.

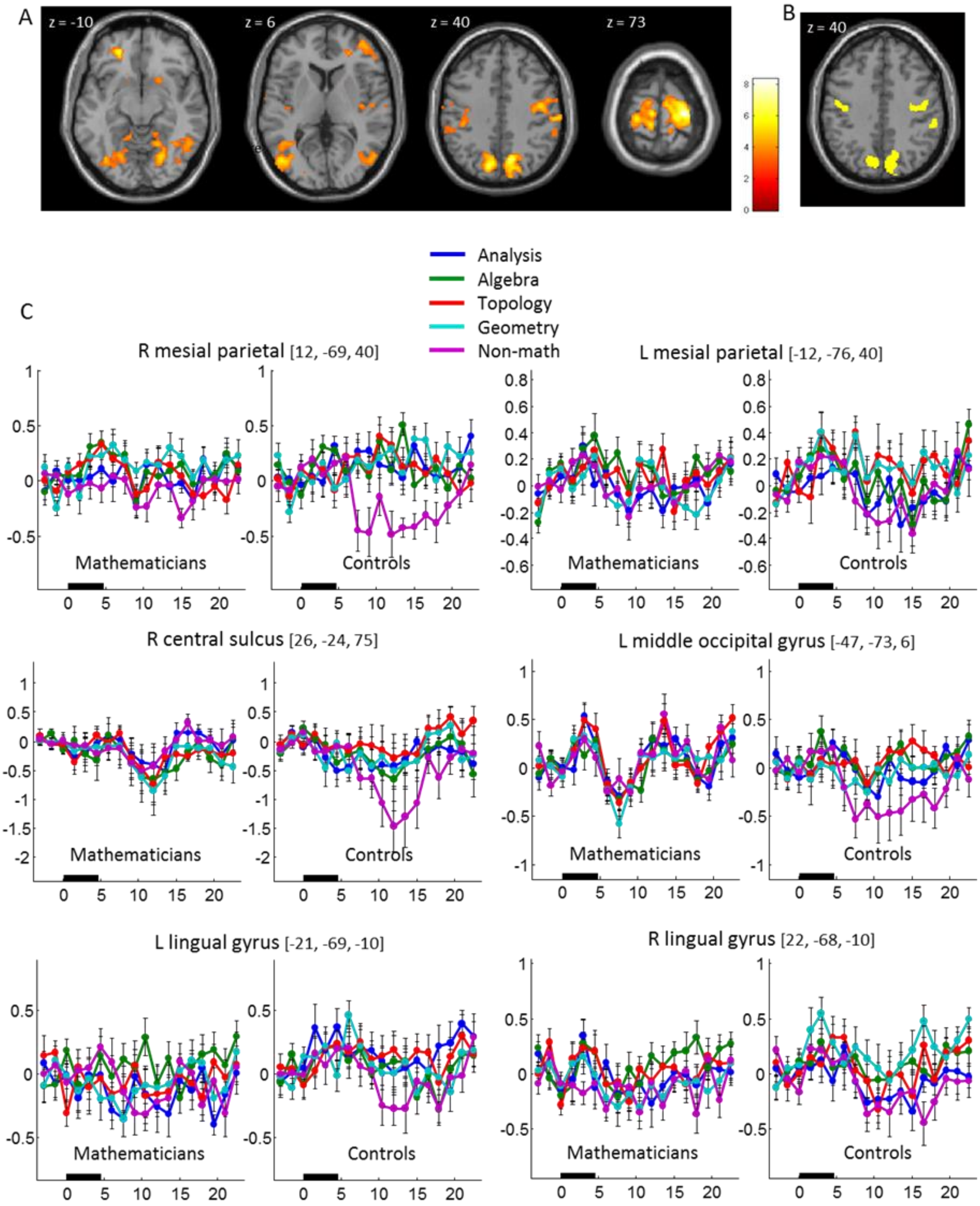

Figure S2. Brain areas showing a difference math > non-math in control subjects. (A) Axial slices showing voxels where activation was higher during reflection on math statements relatively to non-math sentences (voxel $p < 0.001$, cluster $p < 0.05$ corrected for multiple comparisons at the whole-brain level) in control subjects. (B) Slice showing commonalities between the math > non-math contrast and the meaningless > meaningful non-math contrast in control subjects. (C) Plots showing the temporal profile of activation at the main peak of each significantly activated region.

5.3. Variation in brain activation across mathematical problems

Figure 1.3 shows that the majority of the mathematical expertise network was activated jointly by all four mathematical domains, as evidence by an intersection analysis (contrasts of algebra, analysis, geometry and topology, each relative to non-math, in mathematicians during the reflection period; each at p < 0.001; cluster size > 200 voxels). An F-test was used to identify the putative differences between those four contrasts at the whole-brain level. This test revealed significant differences in bilateral parietal posterior regions (peaks at 23, -72, 52; F = 8.39, uncorrected p < 0.001; and at -11, -75, 58; F = 8.73, uncorrected p < 0.001) and left inferior temporal regions (-50, -63, -5; F = 12.01, uncorrected p < 0.001) (figure 1.3A). Examination of the activation profiles, as well as further t-tests, revealed that this pattern was primarily due to a greater activation to geometry problems than to the other three domains combined (at -50, -63, -5, t = 6.39, p < 0.001; at 23, -72, 52, t = 4.39, p<0.001; at -11, -75, 58, t = 4.28, p < 0.001). This contrast also revealed regions showing more activation to geometry than to the other domains of math in bilateral IT, bilateral superior parietal, right intraoccipital sulcus, left supramarginal gyrus, and left inferior parietal cortex. In addition, statements in analysis also induced greater activation than other domains in a mesial frontal orbital region, and statements in topology in the left middle frontal gyrus (table S2, peaks at p < 0.001; cluster size > 200 voxels, corresponding to clusterwise p < 0.05 corrected).

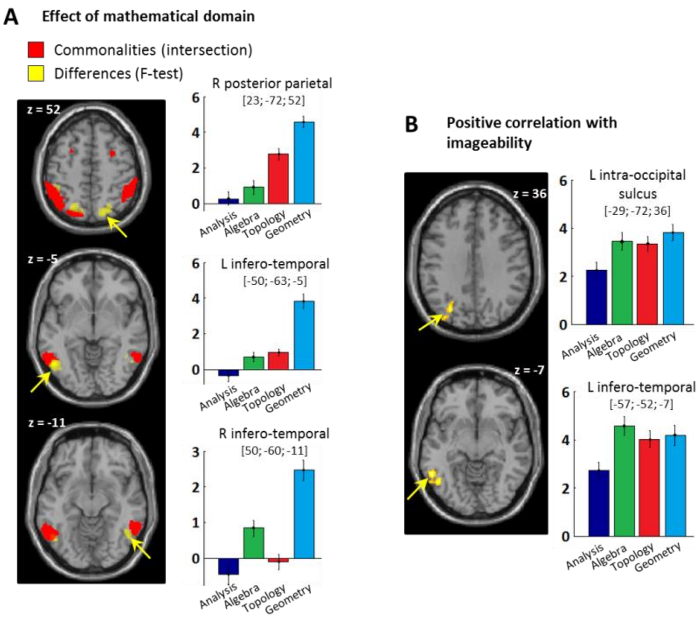

Figure 1.3. Variation in brain activation across mathematical problems. (A) Cortical sites where responses were common (red) or different (yellow) between analysis, algebra, topology, and geometry. The commonalities of the four mathematical domains were assessed by the intersection of activation maps for the contrasts analysis > nonmath, algebra > nonmath, topology > nonmath, and geometry > nonmath (each P < 0.001). Differences in cortical responses across mathematical domains were evaluated by an F-test at the whole-brain level (voxel P < 0.001, cluster P < 0.05 corrected). Bar plots show the activation for each mathematical domain at the principal peaks of three main regions identified in the latter F-contrast (R posterior parietal, L and R infero-temporal). (B) Cortical sites that showed a positive correlation between activation during math reflection and subjective imageability ratings within the meaningful statements in mathematicians.

We also evaluated whether the mathematicians' subjective ratings in the post-MRI questionnaire correlated with brain activity evoked by different mathematical statements. We tested this potential correlation, in mathematicians only, for meaningful math statements, with each of the 6 subjective variables that were rated (comprehension, confidence, difficulty, intuition, immediacy and imageability). Only a single contrast revealed a significant positive correlation between imageability and brain activation, at two sites in the left inferior temporal cortex (peak at -57, -52, -7, T=7.38, $p < 0.001$) and in the left intra-occipital sulcus (peak at -29, -72, 36, $t = 6.06$, $p < 0.001$) (figure 1.3B).

5.4. fMRI activations associated with meaningful mathematical reflection

As a second criterion for brain areas involved in mathematical expertise, we compared the activations during reflection on meaningful versus meaningless mathematical statements. This contrast, which is orthogonal to the previous one and controls for lexical content, fully replicated the results obtained with the contrast of meaningful math > nonmath.

In mathematicians, activation was stronger in bilateral IPS, IT and PFC for meaningful than for meaningless math statements (figure 1.4A; table S1), with the latter inducing only a transient activation in most areas (figure 1.4C, no activation at all in right IPS; figure S3). The same contrast yielded no significant difference in controls, resulting in a significant group X meaningfulness interaction in the same brain regions (figure 1.4B; table S1).

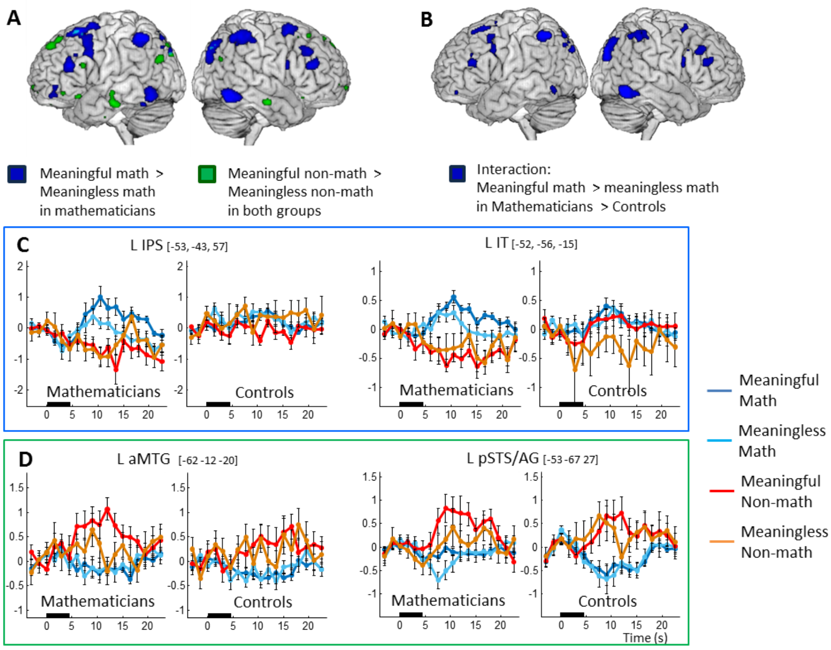

Figure 1.4. Math and nonmath semantic effects. (A) Whole-brain view of semantic effects (meaningful > meaningless) for math statements in professional mathematicians (blue) and for nonmath statements in both groups (green). (B) Mathematical expertise effect: Interaction indicating a large difference between meaningful and meaningless math statements in mathematicians than in controls. (C and D) Average fMRI signals in representative areas responsive to math (C) and to nonmath (D) (see SI Appendix, figure. S3 and S6 for additional areas).

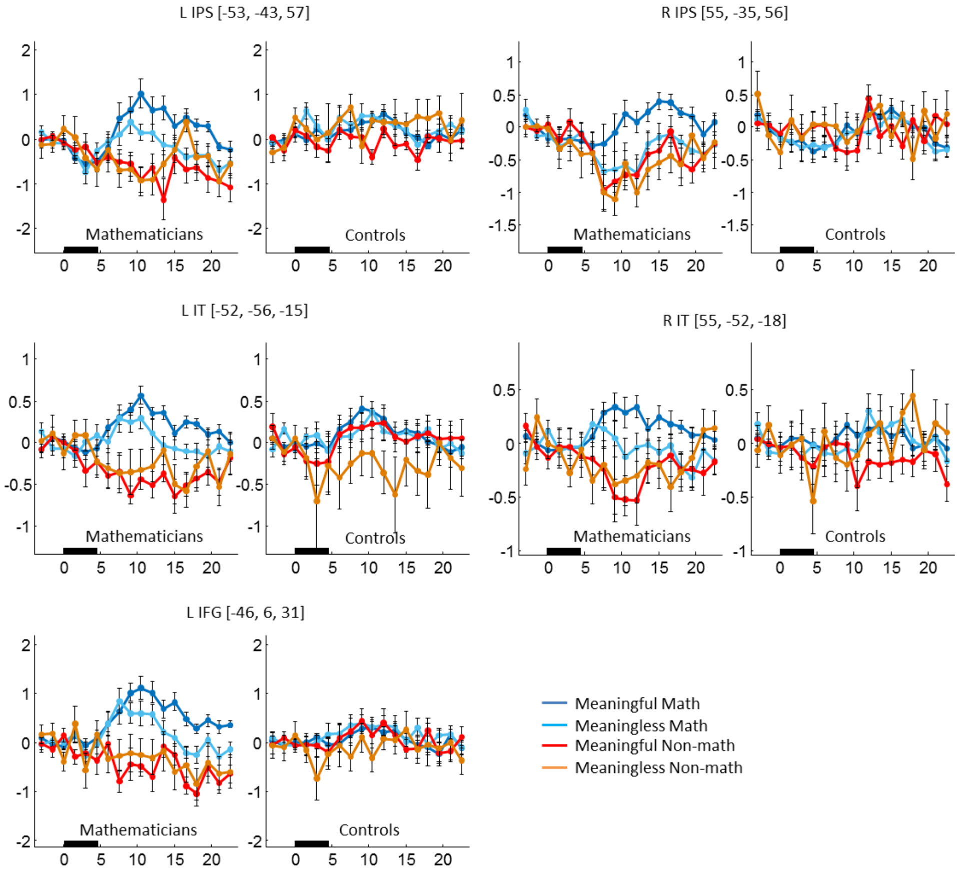

Figure S3. Activation profiles for meaningful and meaningless statements in brain areas responsive to mathematical statements. For both groups, plots at the peaks of the 5 main regions identified in the contrast of math > non-math in mathematicians (same coordinates as figure S1).

5.5. Controls for task difficulty

The activations observed during mathematical reflection overlap with a set of areas which have been termed the "multiple demand system" (Duncan, 2010). Those regions are active during a variety of cognitive tasks that involve executive control and task difficulty (Fedorenko et al., 2013). It is therefore important to evaluate whether our results can be imputed to a greater task difficulty for math relative to non-math statements. As noted in the behavioral section, objective task difficulty, as assessed by percent correct, was not different for math and non-math statements within the mathematicians, and for non-math statements across the two groups of mathematicians and control subjects. However, subjective difficulty, as reported by mathematicians after the fMRI, was judged as slightly higher for the math problems than for the non-math problems (on a subjective scale converted to a 0-100 score: subjective difficulty = 52.4 ± 3.4 for math, and 40.0 ± 4.5 for non-math; $t = 2.4$, $p = 0.03$). Nevertheless, several arguments suggest that this small difference fails to account for our brain-activation results.

First, once the meaningless statements were excluded, difficulty did not differ significantly between meaningful math and non-math statements (subjective difficulty = 53.9 ± 2.8 for meaningful math, versus 49.4 +/- 4.7 for meaningful non-math; t = 0.8, p = 0.5). In other words, the small difference in subjective difficulty (math>non-math) was due only to the greater perceived simplicity of the meaningless general-knowledge statements, whose absurdity was more immediately obvious than that of meaningless math statements. Yet when we excluded the meaningless statements from the fMRI analysis, the difference in brain activation between math and non-math statements remained and was in fact *larger* for meaningful than for meaningless statements (figures 1.2 and 1.4).

Second, to directly evaluate the impact of difficulty on the observed brain networks, within each subject, we sorted the meaningful math and non-math statements into two levels of subjective difficulty (easy or difficult, i.e. below or above that subject's mean of the corresponding category). As expected, the easiest math statements were rated as much easier than the difficult non-math statements (figure 1.5A).

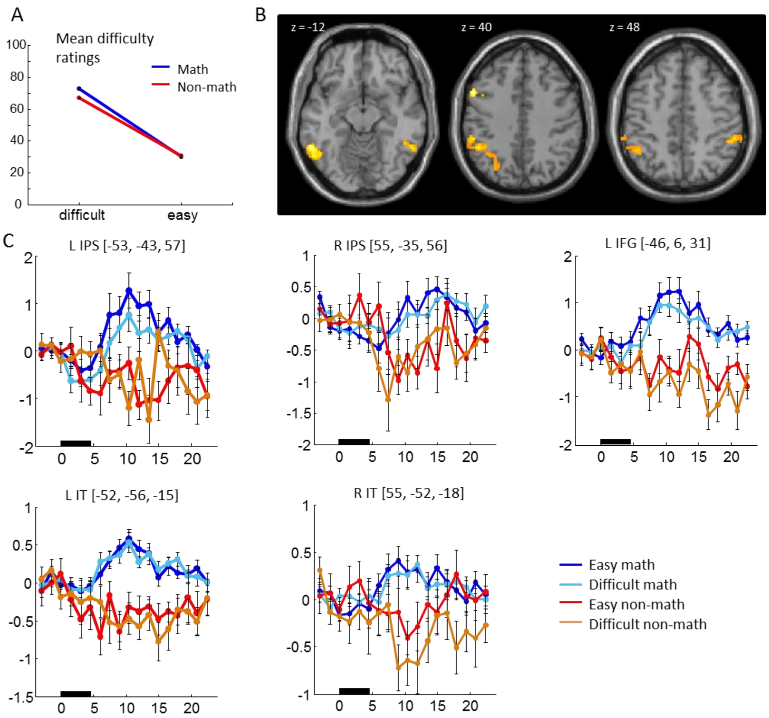

Figure 1.5. Control for task difficulty. For each subject, math and nonmath statements were sorted into two levels of difficulty (easy versus difficult) depending on whether their subjective rating was below or above the subject's mean. (A) Mean difficulty ratings for easy and difficult math and nonmath statements. The results indicate that activation is organized according to domain (math versus nonmath) rather than difficulty. (B) Axial slices showing the principal regions activated in the contrast "easy math > difficult nonmath" in mathematicians across all meaningful problems (voxel P < 0.001, cluster P < 0.05 corrected). This contrast revealed virtually the same sites as the ones that were activated for the standard math > nonmath contrast. (C) Plots report the temporal profile of activation at the principal peaks identified in the contrast of math > nonmath in mathematicians (same coordinates as figure S1).

In spite of this difference, the contrast of meaningful easy math > meaningful difficult non-math again revealed the same sites as those which were activated for the standard math > non-math contrast (figure 1.5B). Thus, those sites were activated even during simple mathematical reflection, and their greater activation for math than for non-math occurred irrespective of task difficulty. Indeed, the time course of fMRI signals in the 5 main regions identified by the math > non-math contrast (figure 1.5C) showed no effect of difficulty. This was confirmed by the contrast of difficult > easy math and difficult > easy non-math which revealed no significant sites. Similar results were obtained when problems were sorted by objective performance (figure S4).

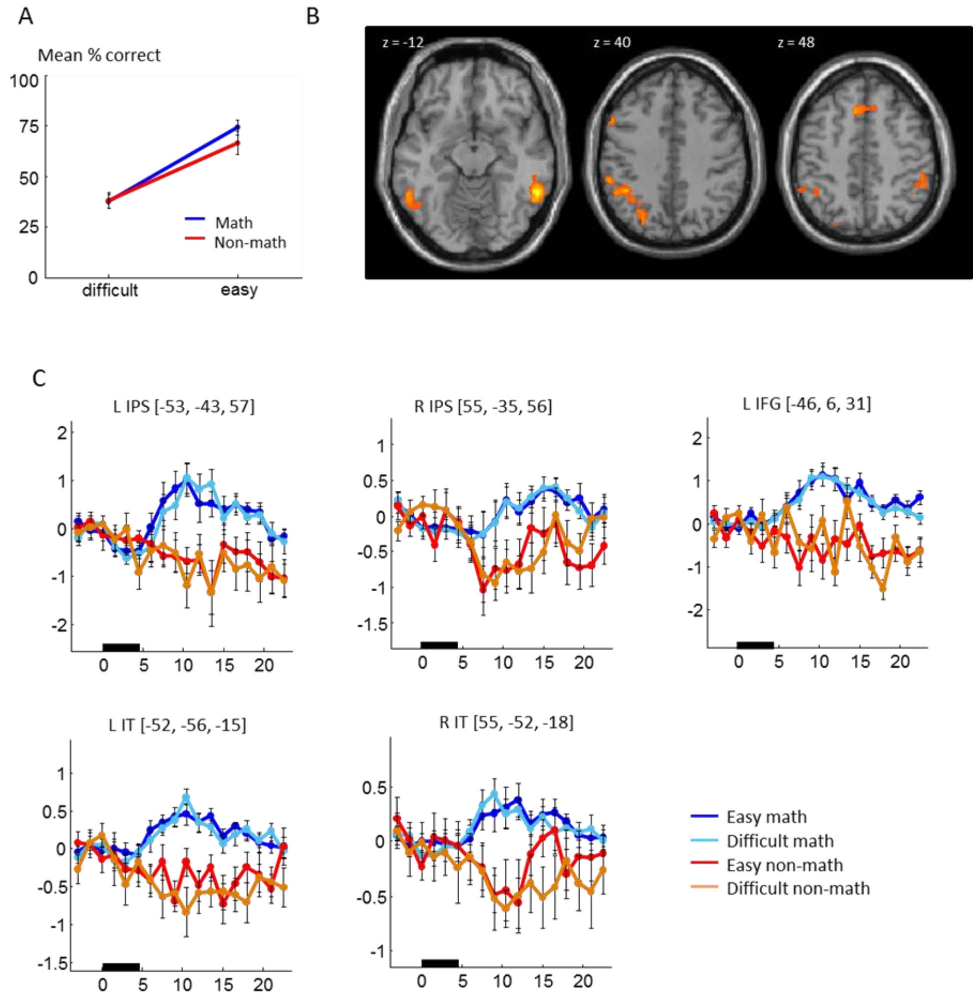

Figure S4. Control for task difficulty. For each subject, math and non-math statements were sorted into two levels of difficulty (easy versus difficult) depending on whether mean performance on a given statement was below or above the global percent correct. (A) Mean correct rates for easy and difficult math and non-math statements. The results again indicate that activation is organized according to domain (math versus non-math) rather than difficulty. (B) Axial slices showing the principal regions activated in the contrast "easy math > difficult non-math" in mathematicians across all meaningful problems (voxel p < 0.001, cluster p < 0.05 corrected for multiple comparisons at the whole-brain level). This contrast revealed virtually the same sites as those which were activated for the standard math > non-math contrast. (C) Plots report the temporal profile of activation at the principal peaks of the 5 main regions identified in the contrast of math > non-math in mathematicians (same coordinates as figure S1).

5.6. Dissociation with the areas activated during non-mathematical reflection

We next examined which regions were activated by non-math statements. Pooling across the two groups, areas activated bilaterally by non-math > math reflection included the inferior angular gyrus (AG, near the temporo/parietal junction), the anterior part of the middle temporal gyrus (aMTG), the ventral inferior frontal gyrus (IFG pars orbitalis, overlapping Brodmann's area 47), an extended sector of mesial prefrontal cortex (PFC; mesial parts of BA 9, 10 and 11) and cerebellum Crus I (figures 1.2A and S5; table S3), consistent with previous studies of semantic networks (Monti et al., 2012; Vandenberghe et al., 1996).

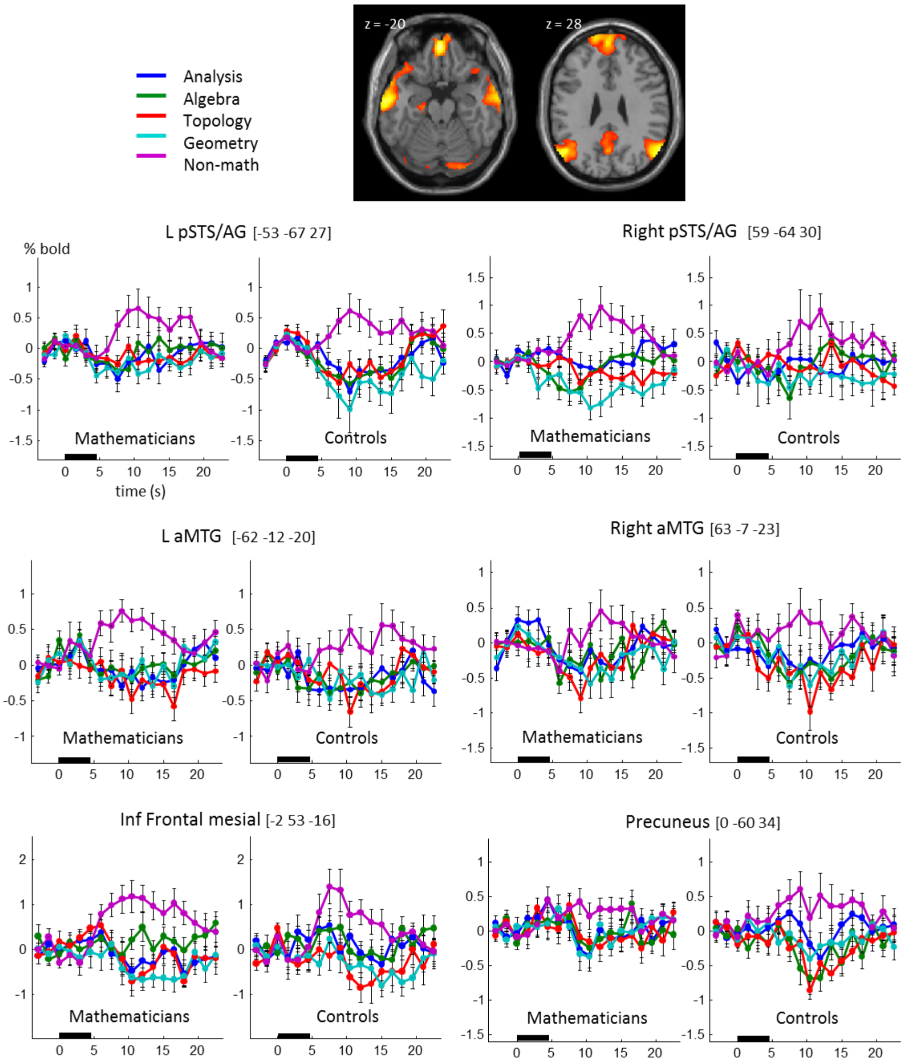

Figure S5. Activation profiles within areas of the general-knowledge network. Axial slices show voxels where activation was higher during reflection on non-math sentences relatively to math statements (voxel $p < 0.001$, cluster $p < 0.05$ corrected for multiple comparisons at the whole-brain level) in control subjects. Plots report the time course of activation at the principal peak of the activated areas.

The majority of these regions showed no difference between groups (table S3). Their time course indicated a significant activation just after non-math statements, and a systematic deactivation to all four types of math statements (figure 1.2D). The contrast meaningful > meaningless non-math statements, which provides an orthogonal means of identifying general-knowledge semantics, pointed to virtually the same sites (figure 1.4A; table S3) and did not differ across groups (figure S6; table S3).

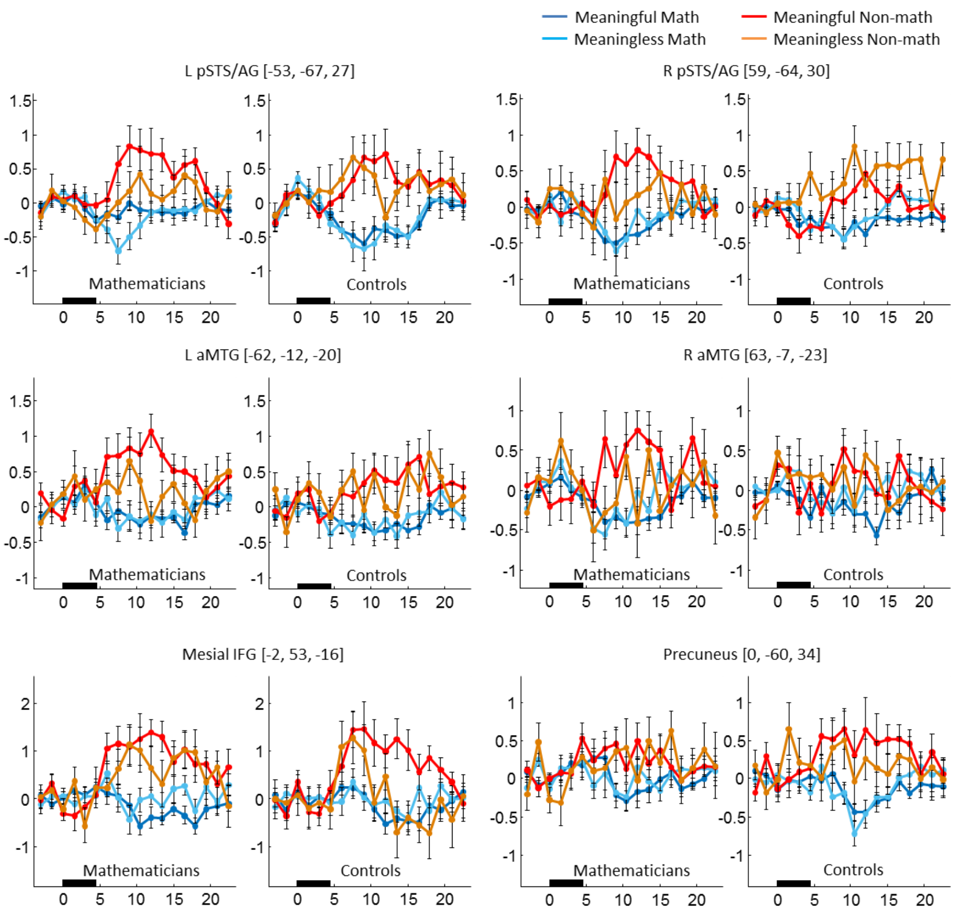

Figure S6. Activation profiles for meaningful and meaningless statements in brain areas mainly responsive to non-mathematical statements during the reflection period. Plots at the peaks of the 6 main regions identified in the contrast of non-math > math in both groups during the reflection period.

Thus, two converging criteria identified a reproducible set of bilateral cortical areas associated with mathematical expertise and that differ from the classical language semantics network. The dissociation, within mathematicians, between the networks for math and non-math was tested formally through the appropriate interactions, i.e. (meaningful - meaningless math) – (meaningful – meaningless non-math) and the opposite contrast (table S4). Stronger activations for meaningful math were again seen in bilateral IT, bilateral IPS, right posterior superior frontal, and left lateral IFG/MFG, while stronger activations for meaningful non-math were in right pSTS/AG, bilateral anterior MTG and ventro-mesial PFC. Crucially, there was essentially no intersection at $p < 0.001$ of the areas for meaningful>meaningless math and for meaningful>meaningless non-math (figure 1.4A, tables S1, S3). The only small area of intersection, suggesting a role in generic reflection and decision-making, was observed outside the classical language network, in bilateral superior frontal (BA 8) and left inferior MFG. Even at a lower threshold ($p < 0.01$ uncorrected), the intersection extended to part of posterior parietal and dorsal PFC but spared perisylvian language cortex.

5.7. Activation profile in language areas

To further probe the contribution of language areas to math, we used a sensitive region-of-interest (ROI) analysis. We selected left-hemispheric regions previously reported (Pallier et al., 2011) as showing a language-related activation proportional to constituent size during sentence processing (temporal pole [TP]; anterior superior temporal sulcus [aSTS]; posterior superior temporal sulcus [pSTS]; temporo-parietal junction [TPj]; inferior frontal gyrus pars orbitalis [IFGorb] and pars triangularis [IFGtri]), plus the left Brodmann area 44 (Amunts et al., 2003). We then used an independent functional localizer (Pinel et al., 2007) to identify subject-specific peaks of activation to sentences (spoken or written) relative to rest, and finally tested the contribution of those language voxels to the main reasoning task.

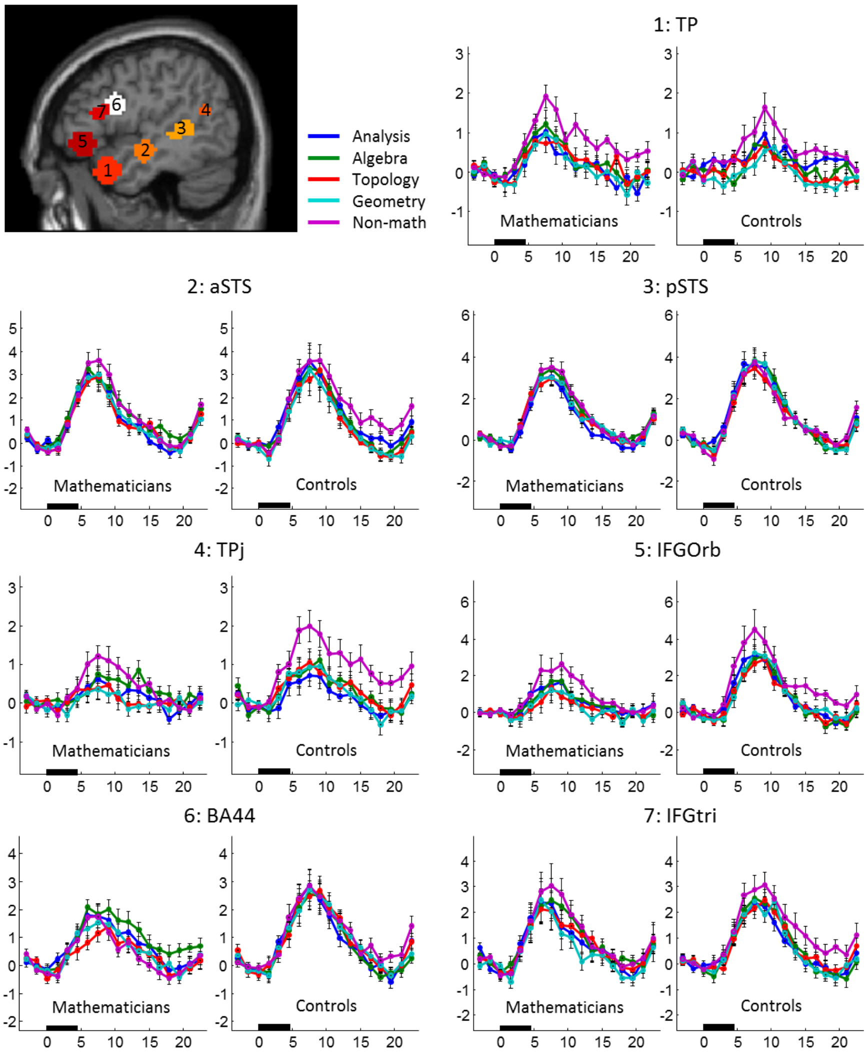

Figure S7. Activation evoked by mathematical and non-mathematical statements in classical language-related regions. The brain slice shows the localization of the seven cortical regions of interest: TP, aSTS, pSTS, TPJ, IFGorb, BA44 and IFGtri. Within each region, plots show the temporal profile of activation for the four domains of math and non-math, averaged across subjects, at the subject-specific peak of activity during an independent localizer for sentence processing. None of these regions appear to be specifically activated during mathematical reflection. On the contrary, several of them show greater activation by non-math than by math statements (see table S5 for statistics).

Figure S7 shows the temporal profile of activation, averaged across participants, at the peak subject-specific voxel, and table S5 presents the corresponding statistics. At this single-voxel level, none of these language regions showed evidence of a contribution to mathematical reflection. In fact, during the reflection period, in mathematicians, TP, pSTS, and IFGOrb responded significantly more to non-math than math. In controls, only aSTS and IFGtri responded more to non-math than to math. We also looked for differences between groups, but the only trends were in the direction of significantly greater activation in controls than in mathematicians (in aSTS and BA44 for non-math statements; and in TP for math statements; uncorrected $p < 0.05$). There was no interaction between group and category in any region. Furthermore, no significant activation was found in those regions for meaningful versus meaningless math statements, neither in mathematicians, nor in controls. However, for meaningful versus meaningless non-math, a significant activation was found in aSTS, and to a lesser extent in pSTS in mathematicians (table S5).

This sensitive ROI approach thus confirmed that language networks do not contribute to mathematical reflection. It could be, however, that these regions have a transient role during the processing of the mathematical statements themselves. We therefore replicated the above analyses with contrasts measuring activation during sentence presentation (table S5, lower part). None of the ROIs were engaged in math listening more than non-math listening, nor in meaningful > meaningless math listening, neither in mathematicians, nor in controls. The only effects were in the converse direction: there was more activation for non-math than for math in aSTS, pSTS, TPJ, IFGOrb, IFGtri and BA44 for mathematicians, and in TPJ and IFGOrb for control subjects. Only IFGOrb showed a group effect, activating less in mathematicians than in controls both during math listening and during non-math listening, without any significant interaction (table S5).

Overall, these results provide no indication that language areas contribute to mathematics, and in fact suggest that, if anything, they activated less for mathematics and/or less in mathematicians.

Whole-brain imaging confirmed a near-complete spatial separation of areas activated by mathematical judgments and by sentence processing (figure S8). A very small area of overlap could be seen in the left dorsal Brodmann area 44 (figure S8B), an area also singled-out in previous reports (Wang et al., 2015) and which should certainly be further investigated in future research. Note, however, that this small overlap was only present in smoothed group images and failed to reach significance in higher-resolution single-subject results (table S5).

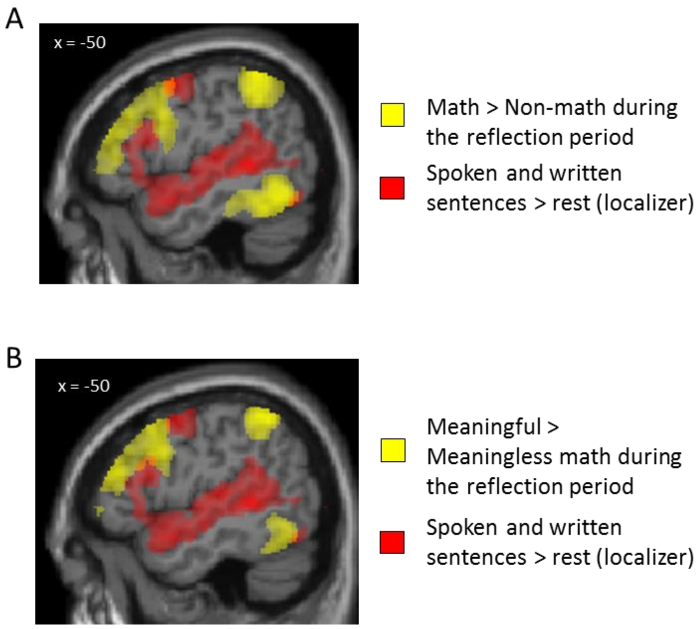

Figure S8. Spatial relationship between the math and language networks. The sagittal slices show, in red, the contrast of spoken and written sentences relatively to rest during an independent functional localizer scan and in yellow, (A) the contrast of math > non-math statements (during the reflection period) and (B) the contrast of meaningful > meaningless math statements (during the reflection period). A very small area of overlap appears in orange in superior frontal cortex mostly in A. The images show how the contours of the math network, in the frontal lobe, spare language-related areas in the left inferior frontal gyrus.

5.8. Relationships between mathematics, calculation, and number detection

We next examined the alternative hypothesis of a systematic relationship between advanced mathematics and core number networks. To this aim, we compared the activations evoked by math versus non-math reflection in mathematicians, with those evoked either by calculation relative to sentence processing (Pinel et al., 2007) or by numbers relative to other visual categories in both mathematicians and controls (after verifying that these groups did not differ significantly on the latter contrasts). Both calculation and simple number processing activated bilateral IPS and IT, thus replicating early observations of number-sense and number-form areas (figure 1.6). Remarkably, those activations overlapped entirely with those activated by higher-level mathematics in mathematicians only (figure 1.6).

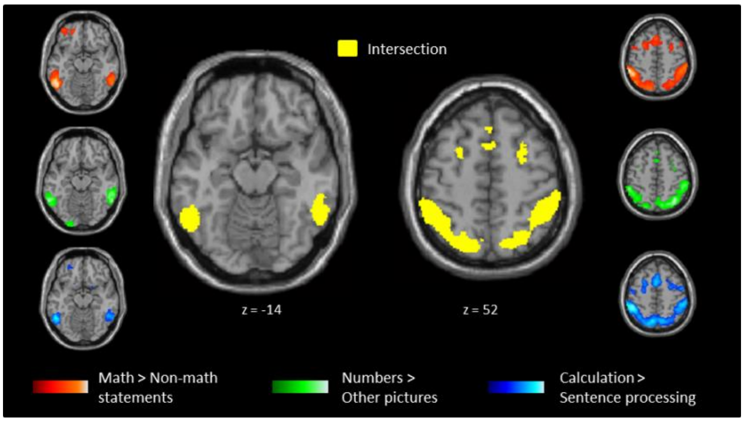

Figure 1.6. Overlap of the mathematical expertise network with areas involved in number recognition and arithmetic. Red, contrast of math versus non-math statements in mathematicians; green, contrast of Arabic numerals versus all other visual stimuli in both mathematicians and controls; blue, contrast of single-digit calculation versus sentence processing in the localizer run, again in both groups; yellow, intersection of those three activation maps (each at $P < 0.001$).

Our mathematical statements carefully avoided any direct mention of numbers or arithmetic facts (see appendix), but some still contained an occasional indirect reference to numbers or to fractions (e.g. $\mathbb{R}^2$, *unit* sphere, *semi*-major axis, etc). We therefore reanalyzed the results after systematic exclusion of such statements. The activation evoked by mathematical reflection remained virtually unchanged (figure S9, table S6). Thus, the overlapping activations to number and to advanced math cannot be explained by a shared component of numerical knowledge, but indicate that high-level mathematics recruits the same brain circuit as basic arithmetic.

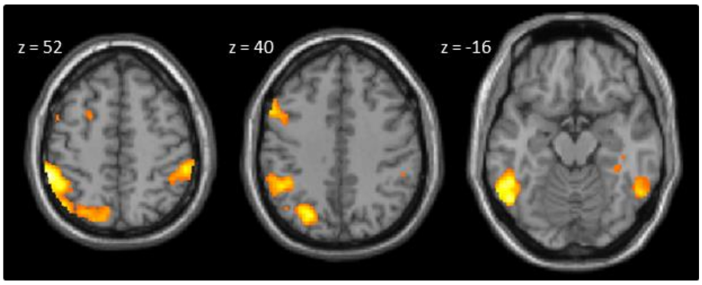

Figure S9. Activation for math > non-math in mathematicians, after removal of sentences containing occasional reference to numbers. Axial slices showing the principal regions activated in the math > non-math contrast in mathematicians, after having removed all statements that contained a reference to numbers. This analysis revealed virtually the same sites as those activated for the overall math > non-math contrast.

Because group-level overlap of activation can arise artificially from inter-subject averaging, we next turned to more sensitive within-subject analyses. First, thanks to independent localizer scans performed in a different cohort of 83 subjects (Pinel et al., 2007), we defined 13 math-related regions in left and right Intraparietal sulci (IPS), infero-temporal cortex (IT), inferior, middle and superior frontal lobes (IFG, MFG, and SFG), mesial supplementary motor area (SMA) and bilateral foci in Cerebellum. In particular within left and right IPS and IT, we verified that the subject-specific voxels activated during simple arithmetic also showed a significant activation during mathematical reflection and during number and formula recognition, and did so more than in the corresponding control conditions (respectively non-math reflection and non-symbolic pictures; table S7).

Second, we used representational similarity analysis to probe whether a similar pattern of activation was evoked, within each subject, by all math-related activities, i.e. mathematical reflection, calculation, and numbers or formula recognition. At subject level, within each of the 13 regions of interest, we computed correlation coefficients between the activations evoked by our main experimental conditions: math and non-math statements, simple calculation and sentence processing, and formulas, numbers, words and non-symbolic pictures. We then compared the correlation of math statements with other math-related condition to the correlation of math statements with the corresponding non-math control condition (figure 1.7). The results revealed that, in all 13 regions, the activation evoked by mathematical reflection was more correlated to the activation evoked by simple calculation than to spoken or written sentence processing (all p*s* < 0.011 uncorrected, table S7). In particular, in bilateral IPS and IT, we first found that the activation topography during the reflection period was more strongly correlated across the four domains of mathematical statements (analysis, algebra, topology and geometry) than between any of those domains and the general-knowledge non-math statements. Second, the activation during mathematical reflection was better correlated with that evoked by simple arithmetical problem solving than with the activation evoked by non-numerical spoken or written sentences in left and right IPS and IT. Third, it was also better correlated with the activation during number recognition (in all four regions) and formula recognition (in left IPS and bilateral IT) than with the activation evoked by non-symbolic pictures or by written words (in bilateral IT only). Similar effects were also observed in other regions: e.g. left IPS, MFG and Cerebellum for formulas or all regions except right Cerebellum for numbers in the comparison with pictures (see table S7). Finally, in bilateral IPS and IT, the activation during simple calculation was better correlated with that evoked by numbers or formulas, than with that evoked by non-symbolic pictures or written words (all *ps* < 0.027 uncorrected, bottom panel of figure 1.7, table S7). Similar correlations with numbers were observed in the other regions except right cerebellum; and left frontal regions also exhibited a stronger correlation with formulas than with pictures (see table S7).

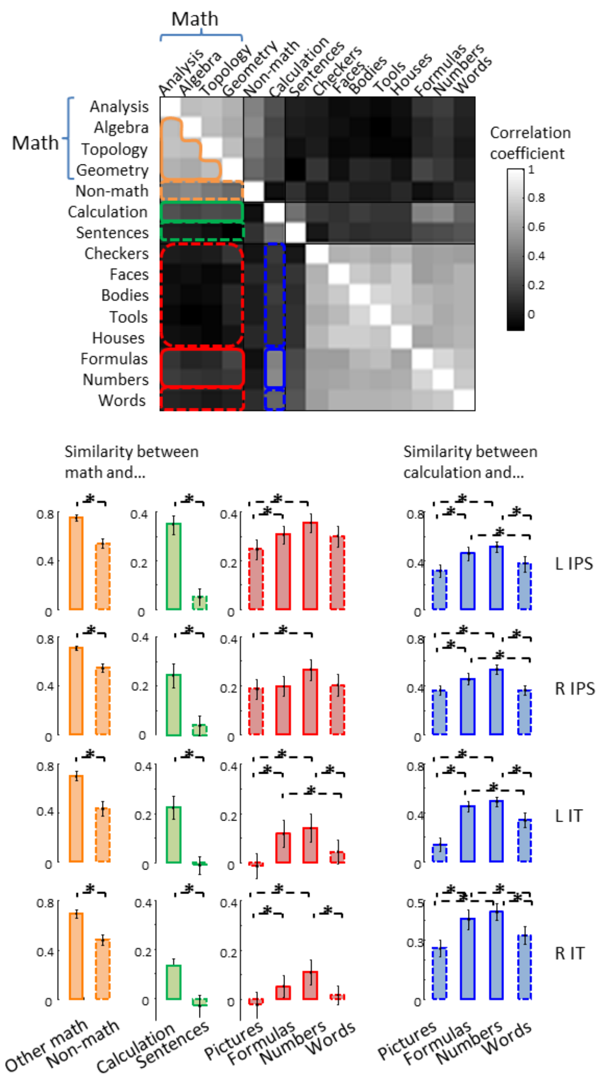

Figure 1.7. Representational similarity analysis. (Top) Sample similarity matrix in left infero-temporal cortex showing the mean, across subjects, of the correlation between the spatial activation patterns evoked by the 15 experimental conditions of the whole experiment: four domains of math plus nonmath presented in auditory runs, calculation and spoken and written sentences from the localizer, and all pictures and symbols tested in visual runs. (Bottom). Mean correlation coefficients are shown in representative regions of interest of the math network. Colors indicate the provenance of the data in the similarity matrix. ROIs (left and right intraparietal sulci and infero-temporal cortices) were defined using a calculation localizer in a different group of subjects. *P < 0.05 (Student t tests). Error bars represent one SEM.

Overall, these high-resolution single-subject analyses confirm that advanced mathematics, basic arithmetic and even the mere viewing of numbers and formulas recruit similar and overlapping cortical sites in mathematically trained individuals.

5.9. Activations during the sentence-listening period

We also analyzed activations during sentence listening, prior to the reflection period. Our conclusions remained largely unchanged. Indeed, in mathematicians, the contrast math > non-math indicated that a subset of the areas involved in math reflection already activated during the auditory presentation of the statements: bilateral IT (-57, -58, -10, t = 10.53; 59, -55, -17, t = 8.42); bilateral IPS (left: -59, -37, 46, t = 7.42 and -29, -73, 37, t = 8.08; right: 39, -61, 54, t = 4.17 and 29, -75, 42, t =

4.88); and bilateral PFC foci (left: -45, 37, 16, t = 7.09 and -48 8 25, t =6.92; right: 51, 7, 24, t = 6.40) (figure S10). Though activation was mostly bilateral, time courses of activation in bilateral intraparietal sulcus suggested that the math network activated early in the left hemisphere and then spread to the right hemisphere (figure S1). Moreover, the bilateral and mesial superior frontal foci that we found activated during reflection were not present during sentence presentation. Conversely, we found an additional activation during sentence presentation in the right head of the caudate nucleus (12, 25, 1, t = 6.79).

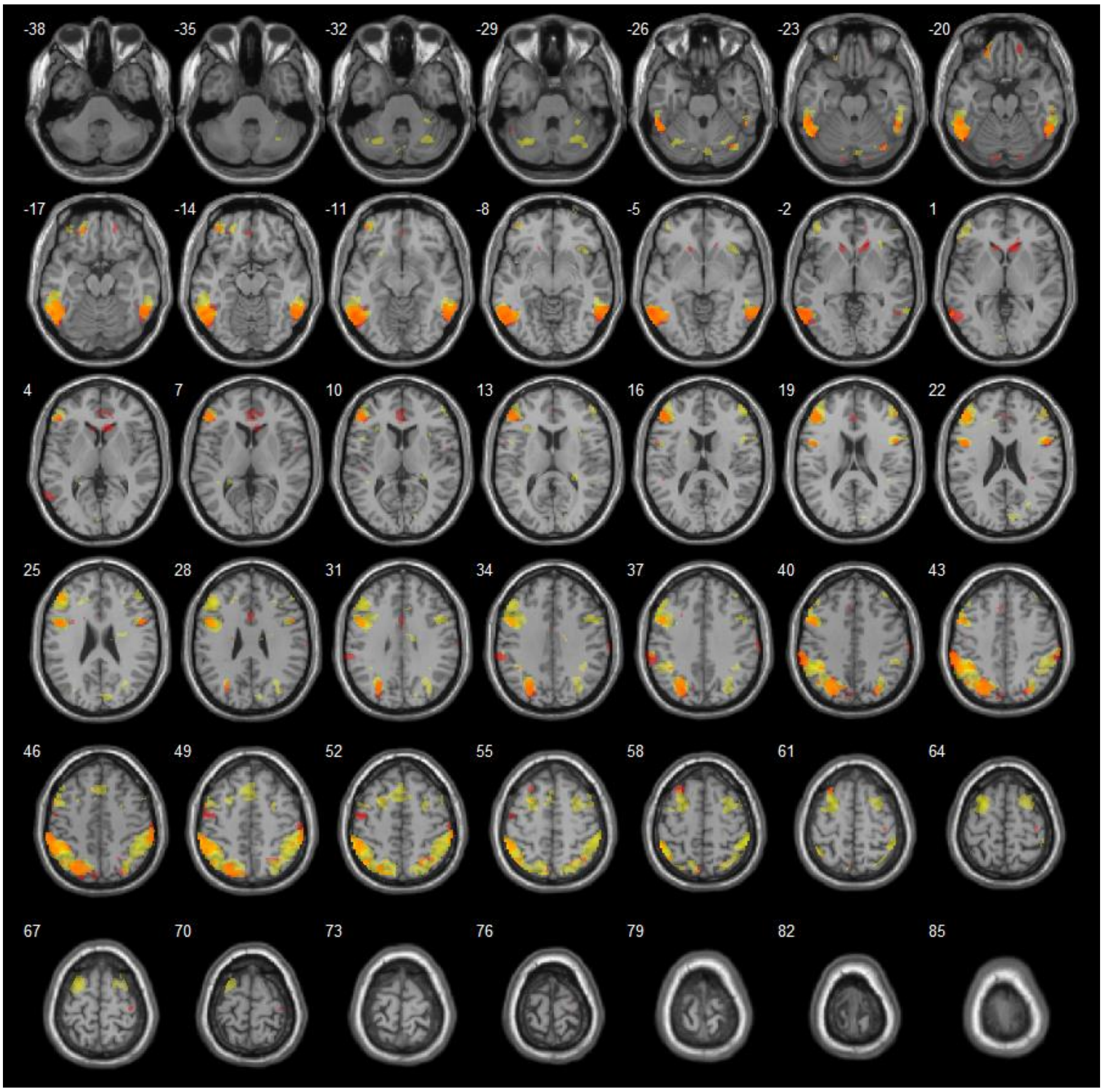

Figure S10. Superposition of the math > non-math contrasts in mathematicians during statement presentation and during the subsequent reflection period. Axial slices show the math > non-math contrasts in mathematicians, separately for activations evoked during sentence presentation in red, and during the reflection period in yellow. The intersection (in orange) reveals that most areas involved in mathematical reflection, particularly in the left hemisphere, were already activated when mathematicians listened to the statements.

For control subjects, the contrast of math > non-math during sentence presentation revealed again a completely different set of areas than the previously identified math network. Some of these areas were found during reflection and thus seemed to activate early, such as the bilateral middle occipital gyri and bilateral insula. Other regions seemed to activate only during sentence presentation. Notably, we found activation in different sub-cortical nuclei including bilateral thalamus (left: -18, -16, 4, t = 5.06; right: 18, -22, 6, t = 5.18), amygdala (left: -29, -6, -26, t = 5.48; right: 27, -1, -28, t = 4.99) and left hippocampus (-39, -30, -10, t = 5.67).

Concerning the non-math statements, the contrast of non-math > math in mathematicians revealed a network that we previously described for non-math > math during the reflection period. We found bilateral temporal activation: anterior MTG (left: -59, -7, -14, t = 10.8; right: 56, -6, -17, t = 9.68), posterior MTG (left: -59, -39, 1, t = 5.52; right: 60, -34, -2, t = 5.55), angular gyrus and temporo-parietal junction (left: -47, -61, 22, t = 10.1; right: 48, -63, 25, t = 6.59). We also found frontal activation: IFGOrb (left: -47, 25, -13, t = 9.28; right: 39, 35, -13, t = 8.11), IFGtri (left: -54, 20, 24, t = 7.79; right: 54, 23, 21, t = 6.06), and mesial frontal sites (superior frontal: -6, 56, 39, t = 8.07; orbitofrontal: -5, 55, -13, t = 5.76). In control subjects, we found additional sites around the calcarine sulcus (-3, -69, 22, t = 6.78), bilateral lingual gyri (left: -15, -57, 3, t = 7.30; right: 12, -49, 3, t = 6.03) and bilateral head of the caudate nucleus (left: -9, 17, -1, t = 5.19; right: 9, 13, -1, t = 5.41).

Two additional effects emerged only during sentence presentation. First, a group X problem type interaction revealed a striking group difference in the bilateral head of the caudate nucleus (figure S11). This region was active in mathematicians only when they were exposed to math statements, and in control subjects only when they were exposed to non-math statements. This effect was confirmed by an examination of the SPM interaction of group and the math > non-math contrast, which was highly significant in the head of the caudate nucleus bilaterally (left: -11, 20, -1, t = 5.95; right 15, 25, -1, t = 7.39), and by plots of temporal profiles of fMRI signals for math and non-math stimuli over the whole regions of interest (figure S11).

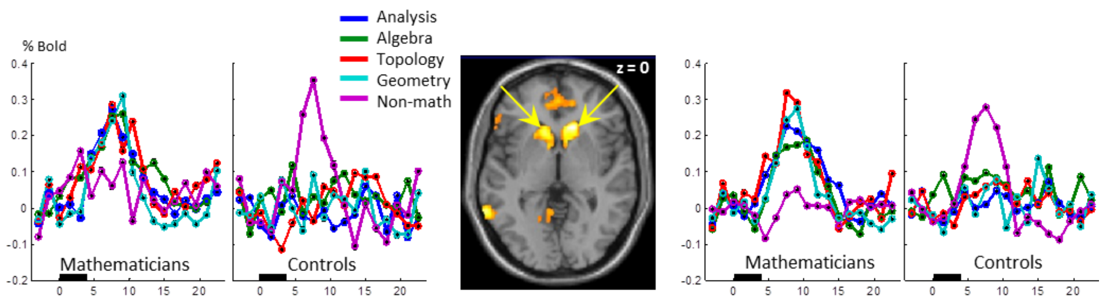

Figure S11. Interaction between group and problem type during statement presentation in the head of the caudate nucleus. The axial slice shows a bilateral activation during statement presentation in the head of the caudate nucleus in the interaction (math>non-math) X (mathematicians – controls) (voxel $p < 0.001$, cluster corrected $p < 0.05$). Plots show the corresponding temporal profile of fMRI signals for the four different domains of math and non-math, separately in mathematicians and control subjects. Signals were averaged across the entire caudate cluster.

The engagement of this subcortical region, which is known to participate in motivation and executive attention, thus shifted radically towards the subject's preferred domain.

Second, another group difference concerned the left angular gyrus. It was deactivated by meaningless compared to meaningful general-knowledge statements in both groups, as previously reported (Pallier et al., 2011; Seghier, 2013). Indeed, studying the contrast of meaningful > meaningless non-math during sentence presentation, the most important cluster was found in the left angular gyrus. It extended to middle occipital gyrus and middle temporal gyrus (in mathematicians: -48, -60, 16, t = 5.28; in controls: -38, -75, 28, t = 4.75; in both groups together: -39, -76, 31, t = 6.12). In mathematicians, it was the only cluster revealed by this contrast. We found additional clusters in control subjects, including three sites exhibiting a significantly greater difference between meaningful and meaningless non-math in controls than in mathematicians: the bilateral middle temporal sulcus (left: -44, -23, -5, t = 5.85; right: 53, -19, 3, t = 4.85), and right Heschl's gyrus (36, -31, 9, t = 4.95). However, in mathematicians only, bilateral angular gyri (left: -48, -60, 16, t = 5.52; right: 44, -79, 22, t = 4.35) also showed a greater activation for meaningful than for meaningless math (figure S12), along with the head of the left caudate nucleus (-14, 19, -2, t = 5.28), some mesial frontal foci (superior frontal: -3, 68, 15, t = 4.95; orbitofrontal: 9, 44, -11, t = 4.28) and middle temporal region (-69, -18, -14, t = 4.74).

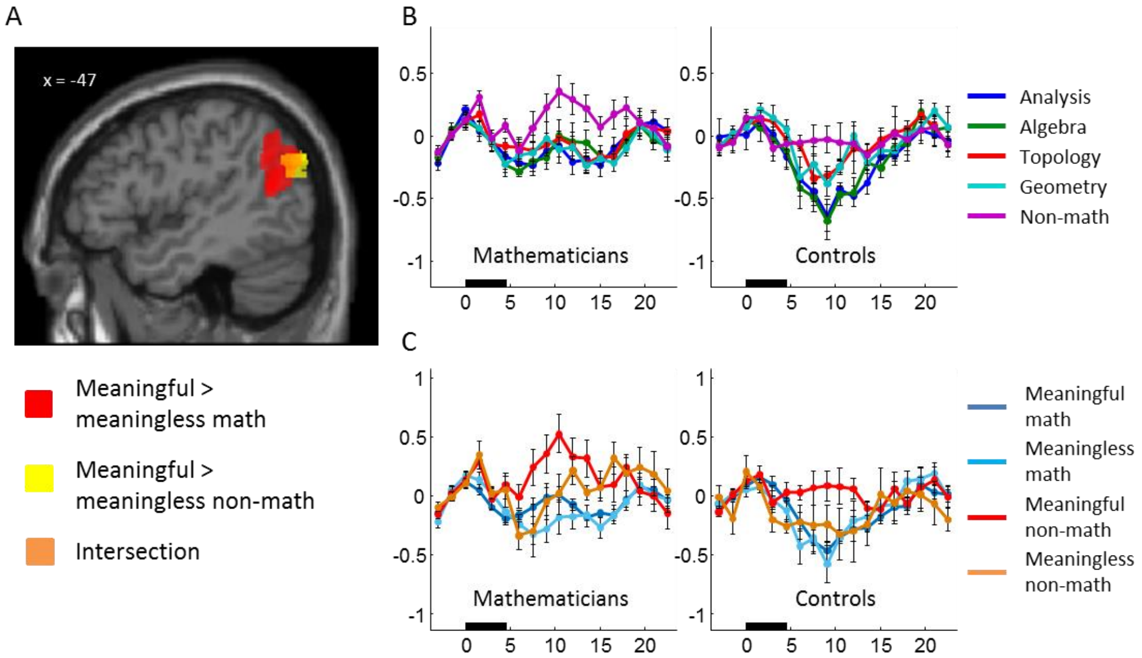

Figure S12. Transient effect of meaningful versus meaningless statements during sentence presentation in the angular gyrus. (A) Sagittal slice centered on the left angular gyrus showing activations to meaningful > meaningless math (in red) and to meaningful > meaningless non-math (in yellow) during sentence presentation (voxel p < 0.001, cluster corrected p < 0.05). The intersection of both contrasts maps appears in orange. (B) Time course of the mean activation within the voxels belonging to the intersection presented in panel A, for the four domains of math and non-math statements in both groups. (C) Time course of the mean activation to meaningful and meaningless math and non-math statements. A transient difference between meaningful and meaningless math is seen only in mathematicians.

Those sites were essentially different from the ones observed during the reflection period, and interestingly, the left angular gyrus appeared in the intersection of meaningful > meaningless contrasts for math and for non-math (figure S12A). In order to clarify the role of this region, we plotted the temporal profiles of the average fMRI signals within that intersection (figure S12B & C).

Such plots revealed that the observed differences occurred in the general context of a deactivation for all mathematical statements relative to baseline, particularly marked in the control subjects. Indeed, we found more deactivation for math in controls than in mathematicians within this region. Moreover, we observed a deactivation for both math and non-math meaningless statements in mathematicians and for all math and meaningless non-math statements in control subjects. In mathematicians, the only group able to distinguish meaningless from meaningful math statements, there was a small transient effect of greater activation to meaningful than to meaningless math. These results therefore suggest that this region is involved in semantic processing of sentences and distinguishes meaningful from meaningless sentences regardless of their mathematical or non-mathematical content. This interpretation fits with previous observations on this area (Humphries et al., 2006; Pallier et al., 2011; Seghier, 2013), which demonstrate an increasing activation in this area in direct proportion to the amount of semantic information available in the stimulus and a systematic deactivation to meaningless materials (e.g. pseudowords or delexicalized “Jabberwocky” sentences), presumably reflecting the contribution of this region to semantic reflection in the resting state. Moreover, mathematical expertise seems to enable the left angular gyrus to extend its function to mathematical statements. Importantly, this is only a transient contribution, restricted to the sentence comprehension period, as this area was deactivated during mathematical reflection.

5.10. Differences between mathematicians and controls in ventral visual cortex

Since high-level mathematics recruits ventral areas of the inferior temporal gyrus involved in the recognition of numbers and expressions, a final question is whether the activation of those regions varies as a function of mathematical expertise. During a one-back task involving the visual presentations of numbers, formulas and other visual stimuli, both mathematicians and controls showed a typical mosaic of ventral occipito-temporal preferences for one category of visual stimuli over all others (figure 1.8A, table S8). Those regions included the right-hemispheric fusiform face area (FFA), bilateral parahippocampal place areas (PPA), bilateral extrastriate body areas (EBA), bilateral lateral occipital cortices for tools (LOC), and left-hemispheric visual word form area (VWFA). Importantly, with high-resolution fMRI, we also found a strong number-related activation in bilateral regions of the inferior temporal gyrus, at sites corresponding to the left and right visual number form areas (VNFA) (Hermes et al., 2015; Shum et al., 2013). We also observed bilateral responses to formulas > other stimuli in both groups at bilateral sites partially overlapping the VNFA. A whole-brain search for interactions with group (mathematicians versus controls) revealed that some of these visual contrasts differed with mathematical expertise. First, the left inferior temporal activation to written mathematical formulas was significantly enhanced in mathematicians relative to controls (-53 -64 -17, $t = 4.27$; figure 1.8B). Single-subject ROI analyses verified that this effect was not simply due to greater variance in anatomical localization in controls compared to mathematicians, but to a genuine increase in the volume of bilateral IT cortex activated by mathematical formulas (table S8). We presume that this region was already present in control subjects because they had received higher education and could therefore recognize basic arithmetic expressions which have been previously related to IT and IPS regions (Maruyama et al., 2012). Just like reading expertise massively enhances the left ventral visual response to written letter strings (Dehaene et al., 2010), mathematical expertise leads to a bilateral enhancement of the visual representation of mathematical symbols.

For numbers, no significant difference between groups was observed using a whole-brain SPM analysis. However, once identified by the overall contrast "number>others", the VNFA peak in the left hemisphere exhibited a small but significant group difference, with more activation in mathematicians than in controls for number > non-symbolic pictures (i.e. excluding formulas and words; $t = 2.31$, $p = 0.028$; no such effect was found at the peak of the right VNFA). Both left and right VNFA also responded more to formulas than to other stimuli in mathematicians relative to controls (left: $t = 3.82$, $p < 0.001$; right: $t = 2.72$, $p = 0.01$; figure 1.8E). Thus, mathematical expertise is associated with a small expansion of number representations in the left VNFA and a bilateral recruitment of the VNFA by mathematical formulas.

Finally, because literacy has been shown to induce a hemispheric shift in face responses (Dehaene et al., 2010), we also examined face processing in our mathematicians. While there was no significant difference between the two groups at the principal peak of the right FFA, a whole-brain search indicated that responses to faces were significantly reduced in mathematicians relative to controls in right-hemispheric IT (44 -45 -17, $t = 4.72$, figure 1.8D). There was also an enhanced response to tools in mathematicians relative to controls in left LOC, just posterior to the activation by formulas (-45 -73 -5, $t = 5.12$, figure 1.8C). These intriguing differences must be considered with caution, as their behavioral impact and causal link to mathematical training remains presently unknown.

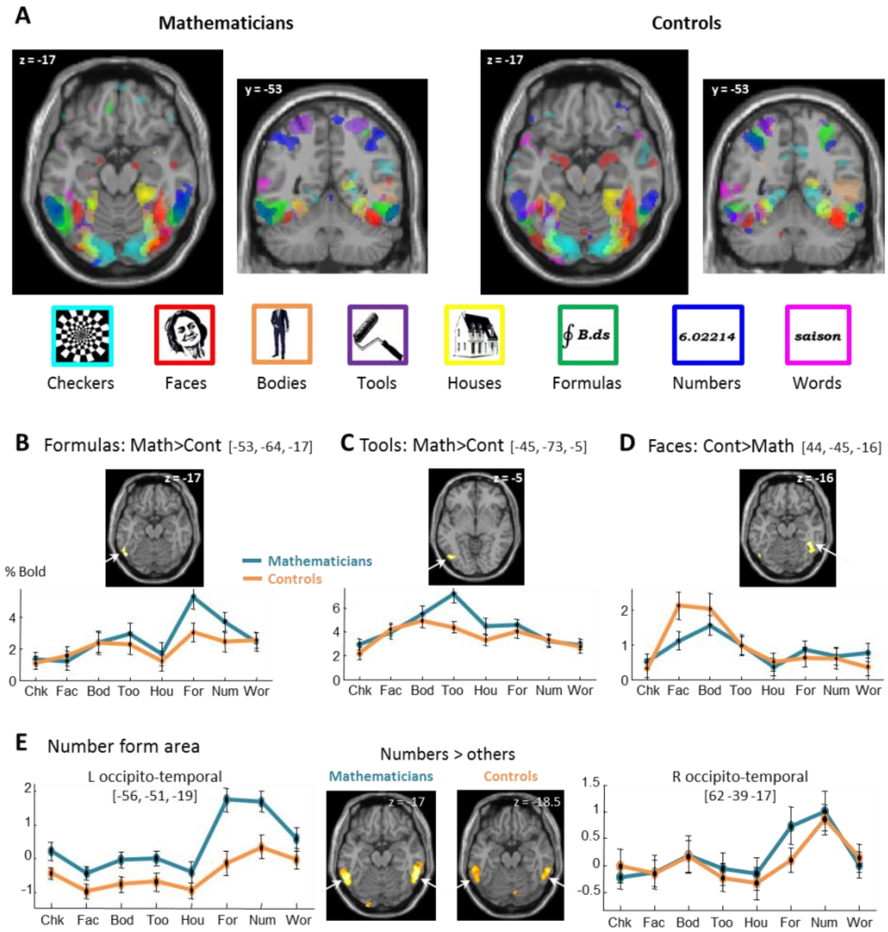

Figure 1.8. Effects of mathematical expertise on the ventral visual pathway. (A) Mosaic of preferences for different visual categories in ventral visual cortex. Slices show the activation for the contrast of a given category (represented by a specific color) minus all others. (B and C) A whole-brain search for larger responses in mathematicians than in controls revealed an effect for formulas in left ventral occipito-temporal cortex (B) and for tools in left lateral occipital cortex (C). Plots show the activation to each category relative to rest at the selected peak for mathematicians and controls. (D) A whole-brain search for smaller responses in mathematicians than in controls revealed an effect for faces in the right fusiform face area (FFA). (E) Slices showing the bilateral visual number form areas (VNFAs) in mathematicians and in controls, assessed by the contrast of numbers minus all other categories. At the peak of the left VNFA, a larger activation was found in mathematicians relative to controls for both numbers and formulas.

6. Discussion

Using high-resolution whole-brain fMRI, we observed the activation of a restricted and consistent network of brain areas whenever mathematicians engaged in high-level mathematical reflection. This network comprised bilateral intraparietal, inferior temporal, and dorsal prefrontal sites. It was activated by all domains of mathematics tested (analysis, algebra, topology and geometry) and even, transiently, by meaningless mathematical statements. It remained silent, however, to non-mathematical statements of matched complexity. Instead, such problems activated distinct bilateral anterior temporal and angular regions.

Our main goal was to explore the relationships between high-level mathematics, language, and core number networks. In mathematicians, we found essentially no overlap of the math-responsive network with the areas activated by sentence comprehension and general semantic knowledge. We observed, however, a strong overlap and within-subject similarity of the math-responsive network with parietal and inferior temporal areas activated during arithmetic calculation and number recognition (table S7). In particular, bilateral ventral inferior temporal areas corresponding to the visual number form area (Hermes et al., 2015; Shum et al., 2013) were activated by high-level mathematics as well as by the mere sight of numbers and mathematical formulas. The latter activations were enhanced in mathematicians. Correspondingly, a reduced activation to faces was seen in the right fusiform gyrus. Those results are analogous to previous findings on literacy, showing that the acquisition of expertise in reading shifts the responses of left ventral visual cortex towards letters and away from faces. (Dehaene et al., 2010; Dundas et al., 2013; Pegado et al., 2014)

Our findings shed light on the roots of mathematical abilities. Some authors argued that mathematics rests on a recent and specifically human ability for language and syntax (Chomsky, 2006), while others hypothesized that it is a cultural construction grounded upon evolutionary ancient representations of space, time and number (Dehaene, 2011; Dillon et al., 2013; Lakoff and Núñez, 2000). In our task, language areas were only activated transiently during the presentation of auditory statements, whether mathematical or non-mathematical. Rather, the activations that we observed during mathematical reflection occurred in areas previously associated with number coding in humans and other animals. Bilateral intraparietal and dorsal prefrontal regions are active during a variety of number-processing and calculation tasks (Dehaene et al., 1999) and contain neurons tuned to numerical quantities (Nieder and Dehaene, 2009). Bilateral inferior temporal regions have been termed “visual number form areas” (VNFA) because they activate to written Arabic numerals much more than to letter strings or other pictures (Hermes et al., 2015; Shum et al., 2013). The VNFAs were previously difficult to detect with fMRI because they lie close to a zone of fMRI signal loss (Shum et al., 2013). However, using a fast high-resolution fMRI sequence that mitigates these difficulties, we found that the VNFAs are easily detectable and are activated bilaterally not only by Arabic numerals, but also by algebraic formulas, arithmetic problems and, in mathematicians only, during high-level mathematical reasoning.

While we only investigated, within our subjects, the relationship between the cortical territories for high-level mathematics, formulas and number processing, previous work strongly suggests that the representation of geometrical relationships and visuo-spatial analogies also calls upon a similar bilateral dorsal prefrontal and intraparietal network (Krawczyk et al., 2011; Watson and Chatterjee, 2012). Indeed, representations of cardinal number, ordinal knowledge, and spatial extent overlap in parietal cortex (Harvey et al., 2015; Prado et al., 2010b). Given those prior findings, our results should not be taken to imply that number is the sole or even the main foundation of higher mathematical abilities; more likely, a complex integration of numerical, ordinal, logical and spatial concepts is involved (Lakoff and Núñez, 2000).

Although one might have thought that the relationship between language and math would depend strongly on the domain of mathematics under consideration, we found no support for this hypothesis. Except for a small additional activation in posterior inferotemporal and posterior parietal cortex for geometry statements, all problems in algebra, analysis, topology and geometry induced correlated and overlapping activations that systematically spared language areas. Using elementary algebraic and arithmetic stimuli, previous fMRI and neuropsychological research in non-mathematicians also revealed a dissociation between mathematical and syntactic knowledge (Klessinger et al., 2007; Maruyama et al., 2012; Monti et al., 2012; Varley et al., 2005). Together, those results are inconsistent with the hypothesis that language syntax plays a specific role in the algebraic abilities of expert adults. Importantly, however, they do not exclude a transient role for these areas in the acquisition of mathematical concepts in children (Spelke, 2003). Imaging studies of the learning process would be needed to resolve this point.

Our results should not be taken to imply that the IPS, IT and PFC areas that activated during mathematical reflection are specific to mathematics. In fact, they coincide with regions previously associated with a « multiple-demand » system (Duncan, 2010) active in many effortful problem-solving tasks (Fedorenko et al., 2013) and dissociable from language-related areas (Fedorenko et al., 2012). Some have suggested that these regions form a "general problem solving" or "general purpose network" active in all effortful cognitive tasks (Hugdahl et al., 2015). Several arguments, however, question the idea that this network is fully domain-general. First, we found no activation of this network during equally difficult reasoning with non-mathematical semantic knowledge. In fact, the easiest mathematical problems caused more activation than the most difficult non-mathematical problems (figure 1.5), and even meaningless mathematical problems caused more activation than meaningful general-knowledge problems (figure 1.4). Second, other studies have found a dissociation between tightly matched conditions of linguistic versus logical or arithmetical problem solving (Monti et al., 2012, 2009). Overall the existing literature suggests that the network we identified engages in a variety of flexible, abstract, and novel reasoning processes that lie at the core of mathematical thinking, while contributing little to other forms of reasoning or problem-solving based on stored linguistic or semantic knowledge.

Our conclusions rest primarily on within-subject comparisons within the group of professional mathematicians (e.g. between math and non-math reasoning, meaningful and meaningless math, etc.). As an additional control, we also presented the same stimuli to a genderand age-matched group of non-mathematically trained but equally talented researchers and professors in humanities and related disciplines. Although mathematicians and controls may still differ on dimensions such as IQ, musical talent, hobbies, etc., such putative differences are irrelevant to our main conclusion of a dissociation between general-knowledge and mathematical reasoning *within* the mathematicians. They also seem unlikely to account for the enhanced ventral visual responses to numbers and math formulas, which most plausibly reflect the much higher frequency with which mathematicians process such symbols.

Previous explorations of the brain mechanisms underlying professional-level mathematics are scarce. One fMRI study scanned 15 professional mathematicians, focusing entirely on their subjective sense of beauty for math expressions (Zeki et al., 2014). The results revealed a medial orbito-frontal correlate for this subjective feeling, but could not determine which brain areas are responsible for the mathematical computations that precede it. The network we observed appears as a plausible candidate that should be tested in further work.

The regions we observe also fit with those showing increased gray matter in mathematicians relative to control subjects of equal academic standing (Aydin et al., 2007). During elementary problem-solving tasks, fronto-parietal activations at locations similar to ours were enhanced in mathematically gifted subjects (Desco et al., 2011). Inter-individual variations in this network predict corresponding variations in fluid intelligence (Duncan, 2010; Gray et al., 2003), which is a major correlate of mathematical skills independently of other language skills. The connectivity between those regions, mediated by the superior longitudinal fasciculus, also increases in the course of normal numerical and mathematical education and in mathematically gifted students relative to others (Emerson and Cantlon, 2012; Matejko and Ansari, 2015; Prescott et al., 2010).

The fact that these brain areas are jointly involved in higher mathematics and basic arithmetic may explain the bidirectional developmental relationships that have been reported between pre-linguistic number skills and later mathematical skills, whereby intuitive number sense predicts subsequent mathematical scores at school (Gilmore et al., 2010; Halberda et al., 2008; Hyde et al., 2014; Starr et al., 2013) and, conversely, mathematical education enhances the precision of the non-verbal approximate number system (Piazza et al., 2013). Educational research also provides ample correlational and interventional evidence suggesting that early visuo-spatial and numerical skills can predict later performance in mathematics. The present results provide a putative brain mechanism through which such links may arise.

7. Supplementary tables

Table S1. Main activation peaks for the math > non-math and the meaningful > meaningless math contrasts.

	Mathematicians								Controls								Mathematicians > Controls							
	Math > Non-math				Meaningful > Meaningless math				Math > Non-math				Meaningful > Meaningless math				Math > Non-math				Meaningful > Meaningless math			
	x	Y	z	t	x	y	z	T	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t
L IPS	-53	-43	57	10.9	-50	-51	52	9.07	-	-	-	-	-	-	-	-	-27	-75	52	7.88	-51	-46	54	6.68
R IPS	50	-36	56	7.30	51	-40	51	7.90	-	-	-	-	-	-	-	-	33	-73	49	5.43	53	-40	51	5.45
L IT	-53	-57	-16	10.4	-56	-58	-16	7.88	-	-	-	-	-	-	-	-	-53	-60	-17	8.26	-62	-57	-10	4.64
R IT	52	-52	-19	7.50	60	-54	-13	9.46	-	-	-	-	-	-	-	-	56	-39	22	5.27	60	-54	-11	7.22
L MFG/ BA46	-44	31	27	7.81	-48	37	22	7.57	-	-	-	-	-	-	-	-	-45	-26	28	7.14	-47	13	36	4.88
L MFG/ BA9	-47	7	31	8.21	-50	10	33	7.33	-	-	-	-	-	-	-	-	-54	14	39	8.57	-53	37	22	5.11
L SFS	-24	8	64	7.11	-26	5	63	7.39	-	-	-	-	-	-	-	-	-27	11	66	7.45	-27	14	60	5.10
R SFS	32	5	56	4.97	30	8	57	9.88	-	-	-	-	-	-	-	-	-	-	-	-	30	8	57	5.79
R MFG/ BA46	50	47	16	6.74	48	38	22	7.60	-	-	-	-	-	-	-	-	-	-	-	-	48	37	22	5.14
R MFG/ BA9 - BA10	50	10	21	6.03	51	11	22	6.61	42	47	25	4.91	-	-	-	-	-	-	-	-	51	11	25	5.45
SMA	-2	23	51	6.12	0	26	49	7.24	-	-	-	-	-	-	-	-	-2	23	51	6.87	-	-	-	-
BA10	-20	47	-16	5.78	-42	55	-13	6.25	-22	44	-10	6.26	-	-	-	-	-	-	-	-	-	-	-	-
L Cereb. 6th lobule	-29	-66	-29	6.00	-3	-81	25	5.22	-	-	-	-	-	-	-	-	-5	-82	-26	6.28	3	-79	-25	4.61
R Cereb. 6th lobule	39	-73	-26	5.24	14	-82	-25	6.03	-	-	-	-	-	-	-	-	8	-81	-23	7.04	8	-78	-28	4.10
L mesial parietal	-	-	-	-	-	-	-	-	-12	76	40	6.50	-	-	-	-	-	-	-	-	-	-	-	-
R mesial parietal	-	-	-	-	-	-	-	-	12	-69	40	6.94	-	-	-	-	-	-	-	-	-	-	-	-
R pre-central sulcus	-	-	-	-	-	-	-	-	26	-24	75	8.34	-	-	-	-	-	-	-	-	-	-	-	-
L post-central sulcus	-	-	-	-	-	-	-	-	-63	0	28	5.85	-	-	-	-	-	-	-	-	-	-	-	-
L MOG	-	-	-	-	-	-	-	-	-47	-73	6	5.50	-	-	-	-	-	-	-	-	-	-	-	-
R MOG	-	-	-	-	-	-	-	-	53	-67	-4	5.56	-	-	-	-	-	-	-	-	-	-	-	-
L Lingual gyrus	-	-	-	-	-	-	-	-	-21	-69	-10	4.50	-	-	-	-	-	-	-	-	-	-	-	-
R Lingual gyrus	-	-	-	-	-	-	-	-	22	-68	-10	5.12	-	-	-	-	-	-	-	-	-	-	-	-
L insula/	-	-	-	-	-	-	-	-	-38	-19	12	5.47	-	-	-	-	-	-	-	-	-	-	-	-

BA13																								
R insula/ BA13	-	-	-	-	-	-	-	-	40	-14	2	4.96	-	-	-	-	-	-	-	-	-	-	-	-
L Putamen	-	-	-	-	-	-	-	-	-14	18	-2	4.86	-	-	-	-	-	-	-	-	-	-	-	-
R Putamen	-	-	-	-	-	-	-	-	18	16	-2	4.85	-	-	-	-	-	-	-	-	-	-	-	-

Table S2. Activation peaks unique to a mathematical domain in mathematicians

Mathematicians	Analysis > other domains				Algebra > other domains				Topology > other domains				Geometry > other domains			
	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t
Mesial frontal orbital	-2	65	-1	4.49	-	-	-	-	-	-	-	-	-	-	-	-
L middle frontal gyrus	-	-	-	-	-	-	-	-	-50	13	27	4.23	-	-	-	-
L inferior temporal	-	-	-	-	-	-	-	-	-	-	-	-	-50	-63	-5	6.39
R inferior temporal	-	-	-	-	-	-	-	-	-	-	-	-	50	-58	-14	5.8
R superior parietal	-	-	-	-	-	-	-	-	-	-	-	-	18	-72	52	5.05
L superior parietal	-	-	-	-	-	-	-	-	-	-	-	-	-23	-66	52	4.94
L supra marginal gyrus	-	-	-	-	-	-	-	-	-	-	-	-	-65	-30	37	4.32
L inferior parietal	-	-	-	-	-	-	-	-	-	-	-	-	-42	-37	42	4.22
R intra occipital sulcus	-	-	-	-	-	-	-	-	-	-	-	-	42	-81	21	5.02

Table S3. Main activation peaks for the non-math > math and the meaningful > meaningless non-math contrasts

	Mathematicians								Controls								Mathematicians > Controls							
	Non-math > Math				Meaningful > Meaningless non-math				Non-math > Math				Meaningful > Meaningless non-math				Non-math > Math				Meaningful > Meaningless non-math			
	x	y	z	t	x	y	z	T	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t
L inferior AG/TP	-56	-70	25	8.30	-	-	-	-	-51	-66	27	8.53	-42	-69	28	4.58	-	-	-	-	-	-	-	-
R inferior AG/TP	60	-64	22	9.83	57	-67	27	4.79	50	-70	33	5.90	41	-66	34	4.01	56	-69	21	5.45	-	-	-	-
L aMTG/ STS	-59	-4	-19	9.16	56	-15	-23	4.69	-63	-7	-10	6.66	-63	-10	-8	5.19	-	-	-	-	-	-	-	-
R aMTG/ STS	60	-9	-25	8.95	-	-	-	-	63	4	-13	5.16	-	-	-	-	60	-7	-25	4.91	-	-	-	-
Precuneus	2	-60	42	6.90	-	-	-	-	-2	-60	34	6.35	-	-	-	-	-	-	-	-	-	-	-	-
L IFGOrb / BA47	-	-	-	-	-51	43	-11	4.95	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
R FGOrb / BA47	-	-	-	-	-	-	-	-	53	25	33	5.39	-	-	-	-	-	-	-	-	-	-	-	-
L SFG	-	-	-	-	-14	43	52	4.96	-18	58	34	7.88	-21	43	48	4.61	-	-	-	-	-	-	-	-
R SFG	-	-	-	-	26	31	57	4.19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Mesial BA 9, 10	0	55	34	7.70	-	-	-	-	2	53	16	5.26	-	-	-	-	-	-	-	-	-	-	-	-
Mesial frontal Orb/ BA 11	3	59	-7	9.52	-8	41	-16	5.20	-2	53	-16	8.46	-6	44	-17	5.37	-	-	-	-	-	-	-	-
L Cereb. Crus I	-18	-88	-29	6.78	-	-	-	-	-6	-84	-25	7.88	-	-	-	-	-	-	-	-	-	-	-	-
R Cereb. Crus I	27	-79	-34	6.11	-	-	-	-	23	-85	-26	9.08	-	-	-	-	-	-	-	-	-	-	-	-
L MOG	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-47	-72	6	4.86	-	-	-	-
R MOG	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	56	-69	21	5.45	-	-	-	-
L para-central /BA4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-15	-31	70	5.04	-	-	-	-
R pre-central	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	26	-24	75	7.21	-	-	-	-
SMA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-18	52	5.04	-	-	-	-
Heschl / Rolandic Oper	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-39	-18	12	4.99	-	-	-	-
Anterior cingulate	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	37	-7	4.39	-	-	-	-

Table S4. Interaction of meaningfulness by math vs. non-math in mathematicians

Mathematicians	Meaningful > Meaningless math - Meaningful > Meaningless non-math				Meaningful > Meaningless non-math - Meaningful > Meaningless math			
	X	y	z	t	x	y	z	t
L Intraparietal sulcus	-62	-34	42	7.78	-	-	-	-
R Intraparietal sulcus	65	-37	46	6.94	-	-	-	-
L inferior temporal	-60	-58	-8	5.00	-	-	-	-
R inferior temporal	59	-57	-10	5.22	-	-	-	-
L lateral IFG/MFG	-44	50	22	5.14	-	-	-	-
R SF sulcus	26	4	55	4.71	-	-	-	-
R pSTS/AG	-	-	-	-	59	-66	27	5.46
L aMTG	-	-	-	-	-57	-15	-11	4.34
R aMTG	-	-	-	-	57	-10	-19	4.64
Mesial frontal Orb	-	-	-	-	2	67	-13	5.4
Mesial superior frontal	-	-	-	-	-14	43	51	4.07

Table S5. Results of regions-of-interest (ROI) analysis in left-hemispheric language regions during reflection.

The table shows the results of contrasts applied to activation from either the reflection period (top) or the sentence presentation period (bottom) of the main task (math/non-math truth value judgment) in voxels isolated in a subject-specific manner, with each ROI, for their responsiveness to spoken or written sentences. A negative sign in the t test indicates an effect in the direction opposite to that indicated in the column title. Significant trends are highlighted in yellow (p<0.05, uncorrected) and in green (p<0.05 with Bonferroni correction for multiple comparisons across the 7 ROIs).

During reflection period																
	Non-math > Math				**Meaningful > Meaningless non-math**				**Meaningful > Meaningless math**				**Controls > Mathematicians**			
	Mathematicians		**Controls**		**Mathematicians**		**Controls**		**Mathematicians**		**Controls**		**During math**		**During non-math**	
	P	t	p	t	p	T	p	t	p	t	p	t	p	t	p	t
TP	0.039	2.29	0.119	1.67	0.272	1.15	0.248	1.21	0.080	-1.90	0.859	0.18	0.039	2.17	0.227	1.24
aSTS	0.082	1.89	0.003	3.53	0.009	3.09	0.669	0.44	0.289	1.10	0.931	0.09	0.114	1.64	0.031	2.27
pSTS	0.001	4.11	0.862	0.18	0.051	2.15	0.068	1.98	0.426	0.82	0.167	1.46	0.378	0.90	0.957	0.05
TPJ	0.080	1.91	0.083	1.95	0.169	1.46	0.458	0.78	0.993	-0.01	0.799	-0.26	0.468	0.74	0.380	0.90
IFGorb	0.024	2.65	0.380	0.91	0.544	0.63	0.442	-0.80	0.313	-1.06	0.578	-0.57	0.386	-0.88	0.254	-1.17
IFGtri	0.289	1.11	0.029	2.46	0.468	0.75	0.568	0.59	0.451	0.78	0.311	1.06	0.955	0.06	0.512	0.67
BA44	0.077	-1.97	0.492	0.71	0.219	1.31	0.807	-0.25	0.111	1.75	0.967	-0.04	0.442	0.78	0.014	2.64

During sentence presentation																
	Non-math > Math				**Meaningful > Meaningless non-math**				**Meaningful > Meaningless math**				**Controls > Mathematicians**			
	Mathematicians		**Controls**		**Mathematicians**		**Controls**		**Mathematicians**		**Controls**		**During math**		**During non-math**	
	P	t	p	t	p	T	p	t	p	t	p	t	p	t	p	t
TP	0.169	1.46	0.141	1.57	0.888	0.14	0.304	-1.07	0.192	-1.38	0.309	1.06	0.090	-1.76	0.286	-1.09
aSTS	0.002	3.98	0.257	1.18	0.087	-1.85	0.671	0.43	0.029	-2.46	0.540	-0.63	0.647	0.46	0.956	0.06
pSTS	0.033	2.38	0.123	1.64	0.123	-1.65	0.096	-1.78	0.354	-0.96	0.693	-0.40	0.486	0.71	0.507	0.67
TPJ	0.013	2.91	0.002	4.21	0.460	0.76	0.267	-1.18	0.071	1.98	0.179	1.46	0.132	1.57	0.173	1.41
IFGorb	0.001	4.79	0.042	2.27	0.439	-0.81	0.092	-1.83	0.325	-1.04	0.898	-0.13	0.045	2.12	0.033	2.27
IFGtri	0.026	2.57	0.568	0.59	0.109	-1.75	0.220	-1.29	0.634	-0.49	0.545	-0.62	0.947	-0.07	0.794	-0.26
BA44	0.046	2.28	0.960	-0.05	0.052	-2.20	0.357	0.95	0.034	-2.45	0.143	1.55	0.185	1.36	0.399	0.86

Table S6. Main peaks for math > non-math and meaningful > meaningless math, after removal of occasional references to numbers, in mathematicians

Mathematicians	Math > Non-math				Meaningful > Meaningless math			
	X	y	z	t	x	y	z	t
L Intraparietal sulcus	-53	-43	57	8	-50	-51	52	7
R Intraparietal sulcus	50	-42	58	5.4	51	-40	52	5.8
L inferior temporal	-56	-49	-19	6.9	-57	-57	-16	7.1
R inferior temporal	53	-51	-19	5.2	60	-58	-13	7.1
L MFG/BA46	-48	39	23	5.6	-49	34	21	5.8
L MFG/BA9	-47	7	31	5.6	-47	18	50	6.3
L SF sulcus	-24	4	64	4.8	-24	4	61	5
R MFG/BA46	-	-	-	-	51	38	21	5.7
R MFG/BA9 - BA10	-	-	-	-	53	11	21	4.4
R SF sulcus	-	-	-	-	30	8	58	7.2
SMA/Frontal Sup mesial	-	-	-	-	-2	28	51	4.8
BA10	-	-	-	-	-41	50	-14	5.3

Table S7. Subject-specific analyses of the relationships between advanced mathematics, simple arithmetic, and number and formula recognition in mathematicians

The top part of the table shows the activations evoked by mathematical reflection, numbers, and mathematical formulas, in subject-specific voxels isolated by their activation during simple arithmetic, within specified regions of interest (ROIs). The bottom part shows, in the same ROIs, comparisons of activation patterns similarity in several math-related stimuli and tasks, versus math and non-math-related stimuli and tasks. Significant trends are highlighted in yellow ($p<0.05$, uncorrected) and in green ($p<0.05$ with Bonferroni correction for multiple comparisons across the 13 ROIs). All approaches indicates that advanced mathematics evokes very similar patterns of activity as simple arithmetic, number recognition, and the recognition of mathematical formulas, particularly in bilateral IPS and IT cortex.

			L Intraparietal sulcus	R Intraparietal sulcus	L inferior temporal	R inferior temporal	L MFG/BA46	L MFG/BA9	L SF sulcus	R MFG/BA46	R MFG/BA9 - BA10	R SF sulcus	SMA/Frontal Sup mesial	L Cerebellum 6th lobule	R Cerebellum 6th lobule
Activation in mathematicians at best peaks for localizer calculation	Math > Non-math reflection	p	0.001	3E-04	0.002	0.009	0.003	1E-04	3E-04	0.006	0.001	0.016	0.063	0.784	0.655
		t	4.10	4.72	3.92	3.04	3.51	5.17	4.75	3.27	3.98	2.73	2.02	0.28	0.46
	Numbers > others	p	0.001	4E-05	0.007	7E-05	0.013	4E-04	0.001	3E-04	0.047	0.004	0.011	0.006	0.115
		t	4.40	5.91	3.14	5.57	2.85	4.64	4.35	4.79	2.18	3.43	2.92	3.28	1.68
	Formulas > others	p	0.018	0.029	0.011	4E-04	0.146	0.026	0.203	0.249	0.469	0.821	0.075	0.919	0.914
		t	2.67	2.43	2.97	4.76	1.55	2.49	1.34	-1.21	-0.75	-0.23	1.95	-0.10	0.11
Statistics on similarity patterns	math*math > math*non-math	p	1.4E-11	3.9E-11	7.0E-10	3.1E-09	3.0E-10	2.6E-08	1.8E-09	9.2E-13	3.3E-10	1.8E-10	5.3E-10	4.5E-13	8.2E-10
		t	19.59	18.19	14.64	13.07	15.59	11.07	13.63	23.96	15.51	16.25	14.95	25.24	14.47
	math*calculation > math*sentence	p	2E-05	1E-04	3.4E-04	0.001	7E-05	7E-06	0.001	0.002	0.001	0.001	4E-04	1E-04	0.011
		t	6.46	5.19	4.71	4.04	5.57	6.92	4.15	3.88	4.05	4.04	4.57	5.23	2.92
	math * formulas > math * non-symbolic pictures	p	0.014	0.301	0.003	0.001	0.003	0.011	0.074	0.651	0.058	0.085	0.077	0.025	0.842
		t	2.82	1.07	3.53	4.18	3.66	2.91	1.93	0.46	-2.06	1.85	1.91	2.50	-0.20
	math * numbers > math * non-symbolic pictures	p	5E-04	0.002	0.001	2E-04	0.002	0.002	4E-04	0.002	0.013	0.029	0.003	0.034	0.072
		t	4.51	3.88	4.06	5.02	3.75	3.81	4.65	3.72	2.84	2.44	3.65	2.34	1.95
	math * formulas > math * words	p	0.807	0.910	0.033	0.179	0.083	0.147	0.292	0.541	0.095	0.273	0.645	0.109	0.228
		t	0.25	-0.11	2.36	1.41	1.87	1.53	1.09	0.63	-1.79	1.14	0.47	1.71	-1.26
	math * numbers > math * words	p	0.062	0.094	0.011	0.021	0.058	0.015	0.006	0.017	0.085	0.129	0.036	0.110	0.669
		t	2.03	1.80	2.91	2.61	2.06	2.77	3.24	2.71	1.85	1.62	2.32	1.71	0.44
	calculation * formulas > calculation * non-symbolic pictures	p	0.001	0.001	2E-06	9E-05	0.006	2E-05	0.006	0.374	0.020	0.930	0.059	0.116	0.427
		t	4.34	4.29	7.88	5.41	3.23	6.20	3.21	0.92	2.62	-0.09	2.06	1.67	0.82
	calculation * numbers > calculation * non-symbolic pictures	p	6E-06	5E-07	3E-07	4E-06	0.002	6E-05	3E-05	0.001	0.001	0.010	0.001	0.014	0.067
		t	6.98	8.70	9.14	7.27	3.86	5.66	6.02	4.23	4.15	3.00	4.37	2.82	1.99

	formulas * (numbers –	p	5E-06	4E-05	7E-05	1E-04	0.002	6E-05	0.010	0.001	0.003	5E-05	8E-07	0.072	0.513
	non-symbolic pictures)	t	7.14	5.93	5.57	5.25	3.73	5.67	2.98	4.10	3.52	5.76	8.36	1.95	0.67
	calculation * formulas >	p	0.029	0.027	0.006	0.041	0.079	0.222	0.236	0.425	0.454	0.074	0.828	0.063	0.298
	calculation * words	t	2.43	2.48	3.22	2.25	1.90	1.28	1.24	0.82	0.77	1.93	-0.22	2.02	1.08
	calculation * numbers >	p	0.003	0.001	0.003	0.002	0.031	0.102	0.015	0.018	0.041	0.002	0.026	0.003	0.091
	calculation * words	t	3.66	4.07	3.55	3.91	2.39	1.75	2.77	2.67	2.25	3.77	2.49	3.62	1.82

Table S8. Volume of activation to different visual stimuli in mathematicians and control subjects

	Principal peaks in both groups				Mathematicians		Controls		Mathematicians > Controls	
	x	y	Z	t	volume (mm^3)	Standard error	volume (mm^3)	Standard error	p	t
L EBA	-50	-76	7	19.1	2846	46	2785	63	0.843	0.20
R EBA	54	-67	3	16.8	2961	45	3055	68	0.768	-0.30
L FFA	-38	-49	-20	10.3	261	14	295	15	0.685	-0.41
R FFA	42	-48	-22	13.4	509	16	521	26	0.918	-0.10
L formulas	-51	-61	-11	11.6	2276	90	1334	63	0.035	2.21
R formulas	55	-55	-17	9.36	803	30	394	22	0.008	2.85
L LOC	-48	-73	-5	9.98	3719	120	2401	141	0.076	1.84
R LOC	50	-70	-7	6.33	1125	62	955	50	0.587	0.55
L PPA	-29	-49	-7	12.4	2739	121	1347	86	0.022	2.42
R PPA	29	-49	-8	13.1	2594	130	2393	132	0.781	0.28
L VNFA	-56	-51	-19	7.94	812	46	591	28	0.303	1.05
R VNFA	62	-39	-17	8.44	643	35	341	19	0.060	1.96
VWFA	-42	-45	-17	4.76	82	6	99	7	0.645	-0.47

Annex: statements used in fMRI experiments

1. Analysis

1.1. True

Statement 1. The Fourier series expansion of a continuous and piecewise C^1 function f converges pointwise to f.

Statement 2. Any real-valued function locally polynomial is polynomial.

Statement 3. The function $\frac{1}{\Gamma(z)}$ admits an analytic continuation to the whole complex plane.

Statement 4. Any compact topological group admits a unique probability measure invariant under left-translations.

Statement 5. The set of test functions is dense in every space L^p, for $p \geq 1$.

Statement 6. A smooth function whose derivatives are all non-negative is analytic.

1.2. False

Statement 7. The spaces $\mathcal{L}^p$ are separable.

Statement 8. The Fourier transform is an isometry from $L^1(\mathbb{R}^n)$ onto itself.

Statement 9. The topological dual of $L^1(\mathbb{R})$ is $L^1(\mathbb{R})$.

Statement 10. An inequality between two functions remains valid for their primitives.

Statement 11. There exists a continuous map from the unit ball into itself without any fixed point.

Statement 12. The distributional derivative of the Heaviside step function is the Heaviside step function.

1.3. Meaningless

Statement 13. Any Dirac's Heaviside function admits a Taylor expansion in L^p.

Statement 14. The space $L^1(\mathbb{R}^n)$ admits a locally polynomial, separable and analytic measure.

Statement 15. In finite measure, the series expansion of the roots of a holomorphic map is reflexive.

Statement 16. The topological dual of a Fourier series admits an analytic continuation.

Statement 17. The trace of the unit ball diverges for some $p \in \{1, \infty\}$.

Statement 18. Any compact polynomial space is isometric to a unique space L^p.

2. Algebra

2.1. True

Statement 19. A square matrix with coefficients in a principal ideal domain is invertible if and only if its determinant is invertible.

Statement 20. For even n, any sub-algebra of $M_n(\mathbb{C})$ of dimension ≤ 4 admits a non-trivial centralizer.

Statement 21. The square matrices with coefficients in a field that are equivalent to a nilpotent matrix are the non-invertible matrices.

Statement 22. Up to conjugacy, there only exist 5 crystallographic groups of the plane.

Statement 23. There exists a 13-dimensional algebra of 4×4-complex matrices.

Statement 24. $\mathbb{Q}$ can be canonically embedded into any field of characteristic zero.

2.2. False

Statement 25. There exists a group of order 169 whose center is reduced to one element.

Statement 26. Any matrix with coefficients in a principal ideal is equivalent to a companion matrix.

Statement 27. A group of which all proper subgroups are abelian is abelian.

Statement 28. In the algebra $M_n(\mathbb{C})$, if two sub-algebras commute, the sum of their dimensions is not greater than n^2.

Statement 29. Any square matrix is equivalent to a permutation matrix.

Statement 30. There exists an infinite order group that admits a finite number of sub-groups.

2.3. Meaningless

Statement 31. Any square invertible ring admits a hexadecimal expansion.

Statement 32. Any matrix with cardinality greater than 3 is factorial.

Statement 33. The field of fractions of an immatricial algebra is embedded in the space of projections.

Statement 34. Any algebra of dimension not greater than 4 is a linear combination of three projections.

Statement 35. There only exist 5 nilpotent canonically additive groups.

Statement 36. The field $\mathbb{R}[i]$ admits a free noetherian centralizer over $\mathbb{Q}$.

3. Topology

3.1. True

Statement 37. A finite left-invariant measure over a compact group is bi-invariant.

Statement 38. The boundary of the Cantor set equals itself.

Statement 39. There exist non-discrete spaces whose connected components are reduced to one point.

Statement 40. The union of a family of pairwise non-disjoint connected subsets of $\mathbb{C}$ is connected.

Statement 41. Any locally finite bounded set of $\mathbb{R}$ is finite.

Statement 42. The quotient of a topological group by its identity component is totally disconnected.

3.2. False

Statement 43. Any continuous bijection between two Hausdorff spaces is a homeomorphism.

Statement 44. There exists a continuous function from the unit sphere onto itself without any fixed point.

Statement 45. Any convex compact set of a Euclidean space is the intersection of a family of closed balls.

Statement 46. In any topological space, every subspace homeomorphic to an open set is also an open set.

Statement 47. Every complete graph can be embedded into the unit sphere of $\mathbb{R}^3$.

Statement 48. Any infinite set of real numbers admits at least one accumulation point.

3.3. Meaningless

Statement 49. Every non-decreasing morphism of the Cantor set is conjugated to a homeomorphism of the unit ball.

Statement 50. Every finite measure on a Hopf algebra is locally modelled on the Haar measure.

Statement 51. The boundary of a homeomorphism has empty interior.

Statement 52. A subset of $\mathbb{C}$ is always left-invariant and right-continuous.

Statement 53. The graph of the completion of a compact group is dense in a partially connected open set.

Statement 54. Every non-countable measure is the intersection of a family of compact groups.

4. Geometry

4.1. True

Statement 55. Any vector field on an even-dimensional sphere vanishes.

Statement 56. The eccentricity of a rectangular hyperbola equals $\sqrt{2}$.

Statement 57. In an ellipse, the ratio of the distance from the center to the directrix equals half the major axe over the eccentricity.

Statement 58. The set of points that are equidistant from two given disjoint lines of $\mathbb{R}^3$ is an hyperbolic paraboloid.

Statement 59. A vector bundle whose base is contractible (for instance, a ball) is trivializable.

Statement 60. The Euclidean orthogonal group has exactly two connected components.

4.2. False

Statement 61. The stereographic projection of the sphere minus one point in the Euclidean space is bounded.

Statement 62. A holomorphic function on a Riemann surface is constant.

Statement 63. Any compact surface is diffeomorphic to an algebraic surface.

Statement 64. At any point P of a directrix of a hyperbola, two tangent lines intersect.

Statement 65. The orthogonal projection of the focus of a parabola on one of its tangent is on the directrix.

Statement 66. Any C^1 vector field on a torus admits a singularity.

4.3. Meaningless

Statement 67. Any Riemannian metric is conjugated to the Haar measure.

Statement 68. The stereographic projection admits $\sqrt{2}$ as Euler characteristic.

Statement 69. The set of points equidistant from two Riemann surfaces is compatible with a paraboloid.

Statement 70. Any holomorphic compact fiber bundle is a particular sphere.

Statement 71. Any variety locally contractible is included in a two-sheeted hyperboloid.

Statement 72. Any locally ellipsoidal submersion is the exponential of a Riemann surface.

5. Non-math

5.1. True

Statement 73. In all Ancient Mediterranean cultures, bulls were considered deities.

Statement 74. In Ancient Greece, a citizen who could not pay his debts was made a slave.

Statement 75. The VAT is a French invention and is a direct consumption tax.

Statement 76. The flag of the Esperanto community is predominantly green.

Statement 77. Apart from the Vatican, Gibraltar is the world's smallest country.

Statement 78. The concept of robots and avatars was already present in Greek mythology.

5.2. False

Statement 79. The Paris metro was built before the Istanbul one.

Statement 80. All borders in Europe, except for Yugoslavia, were set at the end of World War II.

Statement 81. The poet Aragon never joined the Communist party.

Statement 82. The end of the Council of Trent coincides with the fall of the Western Roman Empire.

Statement 83. All members of the Club des Cordeliers were guillotined during the "Terror".

Statement 84. In every society, the market is considered an essential and founding institution.

5.3. Meaningless

Statement 85. The potato flag was guillotined at the end of the Council of Trent.

Statement 86. The institutionalized market drinks Western Roman avatars.

Statement 87. Every indebted green beans have a scientific background.

Statement 88. The Greek mythology is the smallest alcohol derived from the VAT.

Statement 89. Most of the robotic bulls never met Yugoslavia.

Statement 90. A poet is a predominantly green tax over the metro.